Web Appendix to "Semiparametric vector MEM"

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We cover the period between January 1996 and February 2009 for a total of $T = 3261$ observations for each series. We make use of the Oxford Man Institute (OMI) *Realized Library* (Shephard and Sheppard (2010)) data for daily returns and realized kernels for ten important indices together with the range computed using the daily highs and lows downloaded from Datastream. The tickers and names are: DJ30 (Dow Jones Industrials), S&P500 (S&P 500), NASD100 (Nasdaq 100), S&P400 (S&P 400 Midcap), TOS60 (S&P Toronto Stock Exchange), CAC40 (French CAC 40), FTSE100 (UK FTSE 100), DAX30 (German DAX), IBEX35 (Spanish IBEX), NIKKEI225 (Japanese Nikkei 225). All measures are expressed in annualized percentage terms.
This Web Appendix reports extended results commented in the text and those relative to the five tickers non included in the paper (TOS60, CAC40, DAX30, IBEX35 and NIKKEI225).
Further insights from the results can be gained by calculating the Wald test statistics for three joint hypotheses of interest which would strongly characterize the dynamics of the variables involved and the interactions between the two indicators. Relative to the unrestricted model with coefficients (20) in the text, we have

1. *GARCH + HEAVY-RM*, corresponding to a test of $H_0 : \alpha_{12} = 0, \alpha_{21} = 0$;
2. *HEAVY–R + HEAVY-RM*, corresponding to a test of $H_0 : \alpha_{11} = 0, \alpha_{21} = 0$;
3. *GARCH–X + HEAVY-RM*, corresponding to a test of $H_0 : \alpha_{21} = 0$.

We show the p-values of such tests in Table 3.

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Table 1: Unrestricted bivariate vMEM models: estimated coefficients (20) and t–statistics (within parentheses) across various choices of measures and estimation methods (variance measures equation–by–equation, volatility measures equation–by–equation, volatility measures joint estimation).

	$(r_t^2; rv_t^2)$, eq.–by–eq. estimation				$(\|r_t\|; rv_t)$, eq.–by–eq. estimation				$(\|r_t\|; rv_t)$, joint estimation			
Ticker	$\boldsymbol{\alpha}_1$		$\boldsymbol{\beta}_1$		$\boldsymbol{\alpha}_1$		$\boldsymbol{\beta}_1$		$\boldsymbol{\alpha}_1$		$\boldsymbol{\beta}_1$	
DJ30	-0.0398	0.6243	0.6513	-	-0.0558	0.3817	0.6825	-	-0.0593	0.4083	0.6567	-
	(-4.53)	(12.43)	(23.68)		(-3.18)	(7.11)	(14.30)		(-4.01)	(8.87)	(15.88)	
	0.0263	0.3695	-	0.5604	0.0374	0.3568	-	0.5695	0.0379	0.3609	-	0.5642
	(3.47)	(10.66)		(16.01)	(1.86)	(5.66)		(8.31)	(7.20)	(21.93)		(31.64)
S&P500	-0.0281	0.4273	0.7621	-	-0.0851	0.5002	0.5771	-	-0.0672	0.3626	0.7146	-
	(-3.73)	(11.80)	(39.06)		(-4.54)	(8.52)	(10.54)		(-4.81)	(9.27)	(21.17)	
	0.0305	0.3717	-	0.5552	0.0391	0.3557	-	0.5684	0.0407	0.3558	-	0.5663
	(3.99)	(11.41)		(17.28)	(1.95)	(6.12)		(9.06)	(7.23)	(22.00)		(32.57)
NASD100	0.0244	0.4984	0.7255	-	0.0165	0.2650	0.7390	-	0.0140	0.3108	0.6960	-
	(2.59)	(9.94)	(29.72)		(1.03)	(5.26)	(15.92)		(1.04)	(6.94)	(16.74)	
	0.0212	0.4151	-	0.5236	0.0378	0.3845	-	0.5437	0.0396	0.3869	-	0.5387
	(3.99)	(12.71)		(16.14)	(2.30)	(6.69)		(8.70)	(8.56)	(23.96)		(30.66)
S&P400	0.0839	0.2546	0.7956	-	0.0639	0.1281	0.8146	-	0.0676	0.1232	0.8141	-
	(8.68)	(7.86)	(44.13)		(4.59)	(4.25)	(24.47)		(5.95)	(5.28)	(31.17)	
	0.0063	0.3906	-	0.5854	0.0119	0.3568	-	0.6096	0.0132	0.3416	-	0.6244
	(1.98)	(12.11)		(18.80)	(0.98)	(6.23)		(10.12)	(4.17)	(23.48)		(40.71)
TOS60	0.0031	0.5959	0.7064	-	-0.0187	0.3574	0.6980	-	-0.0195	0.3784	0.6777	-
	(0.32)	(8.69)	(23.47)		(-0.93)	(4.92)	(11.64)		(-1.22)	(6.44)	(13.82)	
	0.0226	0.3273	-	0.6167	0.0321	0.3061	-	0.6374	0.0326	0.3094	-	0.6329
	(3.84)	(9.24)		(18.18)	(1.77)	(4.79)		(9.66)	(6.75)	(18.13)		(35.88)

Table 2: Unrestricted bivariate vMEM models: estimated coefficients (20) and t–statistics (within parentheses) across various choices of measures and estimation methods (variance measures equation–by–equation, volatility measures equation–by–equation, volatility measures joint estimation).

	$(r_t^2; rv_t^2)$, eq.–by–eq. estimation				$(\|r_t\|; rv_t)$, eq.–by–eq. estimation				$(\|r_t\|; rv_t)$, joint estimation			
Ticker	$\boldsymbol{\alpha}_1$		$\boldsymbol{\beta}_1$		$\boldsymbol{\alpha}_1$		$\boldsymbol{\beta}_1$		$\boldsymbol{\alpha}_1$		$\boldsymbol{\beta}_1$	
CAC40	-0.0244	0.5517	0.6708	-	-0.0499	0.3603	0.6834	-	-0.0525	0.3722	0.6720	-
	(-2.37)	(10.93)	(24.13)		(-2.58)	(6.29)	(13.33)		(-3.44)	(8.24)	(16.54)	
	0.0407	0.2831	-	0.6373	0.0473	0.2587	-	0.6729	0.0480	0.2687	-	0.6610
	(5.07)	(9.28)		(21.30)	(2.46)	(4.79)		(12.08)	(8.70)	(17.34)		(41.38)
FTSE100	-0.0131	0.6101	0.6511	-	-0.0236	0.3824	0.6585	-	-0.0254	0.3953	0.6471	-
	(-1.13)	(10.28)	(21.14)		(-1.13)	(5.62)	(11.04)		(-1.56)	(7.44)	(13.87)	
	0.0438	0.3306	-	0.5905	0.0406	0.3015	-	0.6380	0.0411	0.3127	-	0.6250
	(4.92)	(9.17)		(17.00)	(1.97)	(4.76)		(9.80)	(7.05)	(17.42)		(33.87)
DAX30	-0.0448	0.5855	0.6106	-	-0.0561	0.3796	0.6359	-	-0.0617	0.4042	0.6106	-
	(-4.51)	(12.07)	(19.72)		(-2.84)	(6.56)	(11.12)		(-3.89)	(8.68)	(13.15)	
	0.0403	0.3727	-	0.5594	0.0474	0.3138	-	0.6210	0.0479	0.3324	-	0.5994
	(4.24)	(10.48)		(16.42)	(2.23)	(5.21)		(10.02)	(8.24)	(20.14)		(35.28)
IBEX35	0.0371	0.4879	0.7095	-	0.0053	0.3157	0.7023	-	0.0021	0.3533	0.6687	-
	(3.58)	(9.20)	(27.53)		(0.30)	(5.12)	(13.27)		(0.14)	(6.65)	(14.56)	
	0.0313	0.3348	-	0.5936	0.0443	0.3029	-	0.6286	0.0451	0.3083	-	0.6217
	(4.89)	(10.17)		(18.59)	(2.55)	(5.10)		(10.29)	(9.46)	(18.95)		(37.16)
NIKKEI225	0.0046	0.4721	0.7684	-	-0.0196	0.3453	0.7099	-	-0.0241	0.3811	0.6783	-
	(0.55)	(9.57)	(35.84)		(-1.15)	(5.75)	(14.23)		(-1.63)	(7.22)	(15.26)	
	0.0210	0.2969	-	0.6418	0.0340	0.2823	-	0.6426	0.0353	0.2960	-	0.6243
	(3.86)	(9.91)		(21.39)	(2.05)	(5.09)		(10.53)	(7.52)	(18.73)		(35.85)

Table 3: P-values of the Wald tests of three restricted specifications (headers *GARCH + HEAVY-RM*, *HEAVY–R + HEAVY-RM*, and *GARCH–X + HEAVY-RM*, respectively) against the unrestricted bivariate vMEM (full $\boldsymbol{\alpha}_1$ and diagonal $\boldsymbol{\beta}_1$). Various choices of measures and estimation methods (variance measures equation–by–equation, volatility measures equation–by–equation, volatility measures joint estimation).

	$(r_t^2; rv_t^2)$, eq.–by–eq. estimation			$(\lvert r_t\rvert; rv_t)$, eq.–by–eq. estimation			$(\lvert r_t\rvert; rv_t)$, joint estimation		
	GARCH	HEAVY-R	GARCH-X	GARCH	HEAVY-R	GARCH-X	GARCH	HEAVY-R	GARCH-X
Ticker	HEAVY-RM	HEAVY-RM	HEAVY-RM	HEAVY-RM	HEAVY-RM	HEAVY-RM	HEAVY-RM	HEAVY-RM	HEAVY-RM
DJ30	0.0000	0.0000	0.0005	0.0000	0.0011	0.0630	0.0000	0.0000	0.0000
S&P500	0.0000	0.0000	0.0001	0.0000	0.0000	0.0511	0.0000	0.0000	0.0000
NASD100	0.0000	0.0000	0.0001	0.0000	0.0412	0.0212	0.0000	0.0000	0.0000
S&P400	0.0000	0.0000	0.0480	0.0001	0.0000	0.3284	0.0000	0.0000	0.0000
TOS60	0.0000	0.0006	0.0001	0.0000	0.1349	0.0763	0.0000	0.0000	0.0000
CAC40	0.0000	0.0000	0.0000	0.0000	0.0017	0.0138	0.0000	0.0000	0.0000
FTSE100	0.0000	0.0000	0.0000	0.0000	0.0763	0.0494	0.0000	0.0000	0.0000
DAX30	0.0000	0.0000	0.0000	0.0000	0.0015	0.0260	0.0000	0.0000	0.0000
IBEX35	0.0000	0.0000	0.0000	0.0000	0.0374	0.0109	0.0000	0.0000	0.0000
NIKKEI225	0.0000	0.0005	0.0001	0.0000	0.0631	0.0401	0.0000	0.0000	0.0000

Table 4: Unrestricted bivariate vMEM: p–values of Ljung-Box test statistics at lags 12, 22, 32 across various choices of measures and estimation methods (variance measures equation–by–equation, volatility measures equation–by–equation, volatility measures joint estimation).

Lag	Ticker	$(r_t^2; rv_t^2)$ eq.–by–eq. estimation	$(\|r_t\|; rv_t)$ eq.–by–eq. estimation	$(\|r_t\|; rv_t)$ joint estimation
12	DJINDUS	0.0589	0.0001	0.0002
	S&PCOMP	0.0086	0.0002	0.0005
	NASA100	0.0000	0.0000	0.0000
	S&PMIDC	0.0000	0.0000	0.0000
	TTOSP60	0.0261	0.0000	0.0001
	FRCAC40	0.0001	0.0000	0.0000
	FTSE100	0.0000	0.0000	0.0000
	DAXINDX	0.0002	0.0000	0.0000
	IBEX35	0.0001	0.0000	0.0000
	JAPDOWA	0.0000	0.0000	0.0000
22	DJINDUS	0.1844	0.0043	0.0054
	S&PCOMP	0.0070	0.0014	0.0024
	NASA100	0.0000	0.0000	0.0000
	S&PMIDC	0.0000	0.0000	0.0000
	TTOSP60	0.1244	0.0002	0.0002
	FRCAC40	0.0052	0.0000	0.0000
	FTSE100	0.0103	0.0000	0.0000
	DAXINDX	0.0018	0.0000	0.0000
	IBEX35	0.0030	0.0000	0.0000
	JAPDOWA	0.0000	0.0000	0.0000
32	DJINDUS	0.3505	0.0137	0.0157
	S&PCOMP	0.0350	0.0027	0.0058
	NASA100	0.0000	0.0000	0.0000
	S&PMIDC	0.0000	0.0000	0.0000
	TTOSP60	0.0003	0.0001	0.0001
	FRCAC40	0.0710	0.0000	0.0000
	FTSE100	0.0743	0.0000	0.0000
	DAXINDX	0.0002	0.0000	0.0000
	IBEX35	0.0490	0.0000	0.0000
	JAPDOWA	0.0001	0.0000	0.0000

Table 5: Bivariate vMEM on *absolute returns* and *realized kernel volatility*. P–values of the Wald test statistics on joint zero restriction hypotheses imposed on the model by column in order to get the model by row. Models are alternative specifications of Equation (9) where a combination of "d", "f" or "0" characterize the $\boldsymbol{\alpha}_1$, $\boldsymbol{\alpha}_2$, $\boldsymbol{\gamma}_1$, $\boldsymbol{\beta}_1$ matrices as *diagonal*, *full*, or zero.

Ticker	$\boldsymbol{\alpha}_1$	$\boldsymbol{\alpha}_2$	$\boldsymbol{\gamma}_1$	$\boldsymbol{\beta}_1$	f d d d	f f d d	f f d f
DJ30	f	0	0	d	0.0000	0.0000	0.0000
	f	d	d	d		0.0022	0.0000
	f	f	d	d			0.0000
S&P500	f	0	0	d	0.0000	0.0000	0.0000
	f	d	d	d		0.0147	0.0000
	f	f	d	d			0.0000
NASD100	f	0	0	d	0.0000	0.0000	0.0000
	f	d	d	d		0.0000	0.0000
	f	f	d	d			0.0000
S&P400	f	0	0	d	0.0000	0.0000	0.0000
	f	d	d	d		0.0000	0.0000
	f	f	d	d			0.0000
TOS60	f	0	0	d	0.0000	0.0000	0.0000
	f	d	d	d		0.0003	0.0000
	f	f	d	d			0.0000
CAC40	f	0	0	d	0.0000	0.0000	0.0000
	f	d	d	d		0.0001	0.0003
	f	f	d	d			0.0001
FTSE100	f	0	0	d	0.0000	0.0000	0.0000
	f	d	d	d		0.0000	0.0000
	f	f	d	d			0.0003
DAX30	f	0	0	d	0.0000	0.0000	0.0000
	f	d	d	d		0.0002	0.0000
	f	f	d	d			0.0098
IBEX35	f	0	0	d	0.0000	0.0000	0.0000
	f	d	d	d		0.0000	0.0000
	f	f	d	d			0.0000
NIKKEI225	f	0	0	d	0.0000	0.0000	0.0000
	f	d	d	d		0.0018	0.0000
	f	f	d	d			0.0002

Table 6: Joint Ljung–Box statistics at different lags for different specifications of a bivariate vMEM on *absolute returns* and *realized kernel volatility*. Matrices included in the specification appear with superscripts (d) when they are *diagonal*, and with (f) when *full*.

Lag	Ticker	$\boldsymbol{\alpha}_1^{(f)}\ \boldsymbol{\beta}_1^{(d)}$	$\boldsymbol{\alpha}_1^{(f)}\ \boldsymbol{\alpha}_2^{(d)}$ $\boldsymbol{\gamma}_1^{(d)}\ \boldsymbol{\beta}_1^{(d)}$	$\boldsymbol{\alpha}_1^{(f)}\ \boldsymbol{\alpha}_2^{(f)}$ $\boldsymbol{\gamma}_1^{(d)}\ \boldsymbol{\beta}_1^{(d)}$	$\boldsymbol{\alpha}_1^{(f)}\ \boldsymbol{\alpha}_2^{(f)}$ $\boldsymbol{\gamma}_1^{(d)}\ \boldsymbol{\beta}_1^{(f)}$
12	DJ30	0.0002	0.0126	0.0755	0.0162
	S&P500	0.0005	0.0532	0.2138	0.0992
	NASD100	0.0000	0.0000	0.0000	0.0015
	S&P400	0.0000	0.0000	0.0000	0.0135
	TOS60	0.0001	0.0016	0.0197	0.0028
	CAC40	0.0000	0.0000	0.0004	0.0001
	FTSE100	0.0000	0.0004	0.0033	0.0018
	DAX30	0.0000	0.0002	0.0042	0.0023
	IBEX35	0.0000	0.0002	0.0433	0.0245
	NIKKEI225	0.0000	0.0008	0.0038	0.0015
22	DJ30	0.0048	0.1215	0.3479	0.1389
	S&P500	0.0021	0.1133	0.3121	0.1076
	NASD100	0.0000	0.0000	0.0019	0.0573
	S&P400	0.0000	0.0000	0.0066	0.2187
	TOS60	0.0002	0.0034	0.0273	0.0110
	CAC40	0.0000	0.0000	0.0003	0.0001
	FTSE100	0.0000	0.0093	0.0413	0.0431
	DAX30	0.0000	0.0001	0.0009	0.0007
	IBEX35	0.0000	0.0001	0.0220	0.0822
	NIKKEI225	0.0000	0.0017	0.0086	0.0096
32	DJ30	0.0130	0.2237	0.4414	0.1690
	S&P500	0.0046	0.1872	0.3546	0.1553
	NASD100	0.0000	0.0000	0.0006	0.0416
	S&P400	0.0000	0.0000	0.0117	0.2535
	TOS60	0.0001	0.0024	0.0138	0.0085
	CAC40	0.0000	0.0000	0.0004	0.0003
	FTSE100	0.0000	0.0122	0.0586	0.0746
	DAX30	0.0000	0.0000	0.0005	0.0006
	IBEX35	0.0000	0.0000	0.0052	0.0381
	NIKKEI225	0.0000	0.0033	0.0180	0.0385

Table 7: Trivariate vMEM *absolute returns* ($|r_t|$), *realized kernel volatility* (RM_t) and *high-low range* (hl_t): joint estimation. Coefficients and t–statistics (within brackets).

Ticker	$\boldsymbol{\alpha}_1$			$\boldsymbol{\alpha}_2$			$\boldsymbol{\gamma}_1$			$\boldsymbol{\beta}_1$		
DJ30	-0.0723	0.5030	-0.0922	0.0447	-	-	0.0324	-	-	0.4270	-0.5225	0.6361
	(-6.77)	(9.84)	(-2.64)	(3.18)			(2.15)			(4.47)	(-4.83)	(7.14)
	0.0108	0.2706	0.0696	-	-0.1577	-	-	0.0532	-	0.0666	0.6760	-
	(2.05)	(43.36)	(5.18)		(-7.09)			(2.51)		(2.63)	(19.82)	
	-	0.3159	-0.0768	-	-	-	-	-	0.0760	-0.3075	-	1.0313
		(9.79)	(-9.61)						(3.61)	(-4.51)		(16.02)
S&P500	-0.1045	0.3956	-0.0559	0.0771	-0.1395	-	0.0464	-	-	0.8132	-	-
	(-12.99)	(6.63)	(-2.63)	(6.18)	(-2.28)		(3.03)			(48.47)		
	0.0166	0.2849	0.0506	0.0226	-0.1587	0.0618	-	0.0751	-	0.3274	0.2795	-
	(2.31)	(35.46)	(2.62)	(2.16)	(-3.97)	(3.19)		(2.94)		(5.12)	(3.14)	
	-	0.3548	-0.0931	-	-0.1934	0.0987	-	-	0.0719	-	-	0.7898
		(9.29)	(-11.86)		(-4.61)	(5.20)			(3.55)			(43.67)
NASD100	-0.0813	0.5035	-	0.0946	-0.2489	-	-	-	-	0.9737	-0.2514	-
	(-5.10)	(8.06)		(5.89)	(-2.96)					(127.80)	(-6.95)	
	0.0490	0.4384	-	-0.0467	-0.2858	-	-	-	-	-0.0092	0.8486	-
	(10.43)	(24.54)		(-10.01)	(-11.79)					(-2.39)	(55.63)	
	-	0.4019	0.0351	-	-0.2028	-	-	-	-	-	-0.2074	0.9619
		(6.99)	(6.62)		(-2.64)						(-5.60)	(166.69)
S&P500	0.0012	0.3422	-	-	-	0.0470	-	-	-	0.9245	-0.3336	-
	(0.23)	(10.40)				(3.77)				(69.29)	(-9.83)	
	-	0.3151	0.0652	-	-0.1320	-0.0552	-	-	-	-0.0224	0.8209	-
		(14.78)	(7.98)		(-5.00)	(-7.90)				(-4.33)	(49.49)	
	-	0.3517	0.0692	-	-0.1105	-	-	-	-	-	-0.2112	0.8901
		(9.39)	(6.42)		(-2.75)						(-6.52)	(57.66)
TOS60	-0.0405	0.2559	0.0849	-	-	-	-	-	-	0.7127	-	-
	(-2.71)	(4.80)	(2.57)							(18.29)		
	0.0252	0.2993	0.0510	-0.0227	-0.1611	-	-	-	-	-	0.9740	-0.1519
	(4.93)	(12.79)	(5.67)	(-4.57)	(-5.49)						(128.52)	(-8.57)
	-	0.3122	0.0950	-	-0.1253	-	-	-	-	-	-	0.7416
		(7.84)	(6.42)		(-3.21)							(31.79)

Table 8: Trivariate vMEM *absolute returns* ($|r_t|$), *realized kernel volatility* (RM_t) and *high-low range* (hl_t): joint estimation. Coefficients and t–statistics (within brackets).

Ticker	$\boldsymbol{\alpha}_1$			$\boldsymbol{\alpha}_2$			$\boldsymbol{\gamma}_1$			$\boldsymbol{\beta}_1$		
CAC40	-0.0635	0.3047	-	0.0350	-	-	-	-	-	0.7201	-	-
	(-4.19)	(8.19)		(2.33)						(22.54)		
	0.0416	0.2479	0.0719	-	-0.1589	-	-	-	-	0.1063	0.6717	-
	(9.77)	(10.04)	(5.11)		(-6.22)					(3.90)	(19.07)	
	-	0.2250	0.0573	0.0120	-0.0988	-	-	-	-	-	-	0.7969
		(7.07)	(3.83)	(1.99)	(-3.19)							(41.28)
FTSE100	-0.0470	0.3594	-	0.0741	-	-	-	-	-	0.8258	-0.2154	-
	(-3.25)	(9.10)		(4.32)						(24.36)	(-4.89)	
	0.0163	0.2984	0.0838	-	-0.1757	-0.0592	-	-	-	-	0.9772	-0.1322
	(3.95)	(11.74)	(5.89)		(-5.92)	(-4.83)					(149.04)	(-8.14)
	-	0.3333	0.0487	0.0277	-0.1504	-	-	-	-	-	-	0.7573
		(9.64)	(3.71)	(3.87)	(-4.36)							(33.09)
DAX30	-0.0648	0.3306	-	0.0706	-0.1984	0.0729	-	-	-	0.8478	-	-0.0769
	(-3.80)	(6.10)		(4.52)	(-3.38)	(3.81)				(28.16)		(-3.87)
	0.0499	0.2979	0.0595	-0.0442	-0.1997	-	-	-	-	-0.0896	0.9757	-0.0615
	(7.36)	(13.83)	(5.56)	(-6.10)	(-7.46)					(-3.97)	(78.49)	(-5.02)
	0.0217	0.1823	0.0702	-0.0211	-0.1434	0.0719	-	-	-	-0.0473	-	0.8551
	(2.58)	(7.15)	(3.82)	(-2.62)	(-4.95)	(3.88)				(-2.52)		(61.54)
IBEX35	-0.0456	0.2577	0.1535	0.0857	-	-0.1724	-	-	-	0.9325	-0.2182	-
	(-2.41)	(7.52)	(3.62)	(4.76)		(-4.26)				(41.84)	(-5.23)	
	0.0404	0.2717	0.0730	-0.0230	-0.1439	-0.0293	-	-	-	-	0.9207	-0.1118
	(7.47)	(10.95)	(5.46)	(-4.79)	(-5.21)	(-2.37)					(54.67)	(-6.07)
	0.0266	0.2484	0.0887	-	-0.1345	-	-	-	-	-	-	0.7739
	(3.93)	(6.66)	(5.66)		(-3.58)							(37.30)
NIKKEI225	-0.0770	0.3390	-	0.0808	-0.1810	0.1205	-	-	-	0.9936	-	-0.2845
	(-5.40)	(5.13)		(5.62)	(-2.34)	(5.04)				(308.99)		(-8.03)
	0.0122	0.1942	0.1402	-0.0112	-0.1025	-0.0823	-	-	-	-	0.9684	-0.1253
	(2.66)	(7.37)	(8.79)	(-2.45)	(-3.65)	(-6.09)					(88.23)	(-7.61)
	-	0.1974	0.0951	-	-0.0881	-	-	-	-	-	-	0.7888
		(5.02)	(5.90)		(-2.24)							(37.42)

Table 9: P–values of the joint Ljung-Box statistics at different lags on four specifications of a trivariate vMEM on *absolute returns*, *realized kernel volatility* and *high-low range*. Matrices included in the specification appear with superscripts (d), when they are *diagonal* and with (f) when *full*).

Lag	Ticker	$\boldsymbol{\alpha}_1^{(f)}$ $\boldsymbol{\beta}_1^{(d)}$	$\boldsymbol{\alpha}_1^{(f)}$ $\boldsymbol{\alpha}_2^{(d)}$ $\boldsymbol{\gamma}_1^{(d)}$ $\boldsymbol{\beta}_1^{(d)}$	$\boldsymbol{\alpha}_1^{(f)}$ $\boldsymbol{\alpha}_2^{(f)}$ $\boldsymbol{\gamma}_1^{(d)}$ $\boldsymbol{\beta}_1^{(d)}$	$\boldsymbol{\alpha}_1^{(f)}$ $\boldsymbol{\alpha}_2^{(f)}$ $\boldsymbol{\gamma}_1^{(d)}$ $\boldsymbol{\beta}_1^{(f)}$
12	DJ30	0.0000	0.0005	0.0166	0.1074
	S&P500	0.0000	0.0002	0.0680	0.1887
	NASD100	0.0000	0.0000	0.0000	0.0001
	S&P400	0.0000	0.0000	0.0001	0.0156
	TOS60	0.0000	0.0000	0.0414	0.1554
	CAC40	0.0000	0.0000	0.0042	0.0057
	FTSE100	0.0000	0.0000	0.0881	0.0743
	DAX30	0.0000	0.0000	0.0000	0.0011
	IBEX35	0.0000	0.0000	0.0504	0.0417
	NIKKEI225	0.0000	0.0000	0.0094	0.0200
22	DJ30	0.0005	0.0252	0.1972	0.4067
	S&P500	0.0000	0.0003	0.1020	0.0877
	NASD100	0.0000	0.0000	0.0000	0.0000
	S&P400	0.0000	0.0000	0.0181	0.2376
	TOS60	0.0000	0.0001	0.1030	0.4377
	CAC40	0.0000	0.0000	0.0020	0.0032
	FTSE100	0.0000	0.0004	0.1475	0.2672
	DAX30	0.0000	0.0000	0.0000	0.0003
	IBEX35	0.0000	0.0000	0.0801	0.2637
	NIKKEI225	0.0000	0.0000	0.0071	0.0592
32	DJ30	0.0005	0.0213	0.1331	0.3197
	S&P500	0.0000	0.0005	0.0940	0.1088
	NASD100	0.0000	0.0000	0.0000	0.0002
	S&P400	0.0000	0.0000	0.0885	0.5170
	TOS60	0.0000	0.0000	0.0062	0.0654
	CAC40	0.0000	0.0000	0.0036	0.0036
	FTSE100	0.0000	0.0001	0.0638	0.2116
	DAX30	0.0000	0.0000	0.0000	0.0025
	IBEX35	0.0000	0.0000	0.0217	0.0914
	NIKKEI225	0.0000	0.0000	0.0011	0.0522

Table 10: P-values of the non–causality tests for a trivariate vMEM on *absolute returns*, *realized kernel volatility* and *high-low range*. The specification is $\boldsymbol{\alpha}_1^{(f)}$, $\boldsymbol{\alpha}_2^{(f)}$, $\boldsymbol{\gamma}_1^{(d)}$ and $\boldsymbol{\beta}_1^{(f)}$.

Null hypothesis	DJ30	S&P500	NASD100	S&P400	TOS60
$\lvert r\rvert \nleftarrow rv$	0.0000	0.0000	0.0000	0.0000	0.0000
$\lvert r\rvert \nleftarrow hl$	0.0000	0.0086	0.0157	0.0002	0.0100
$rv \nleftarrow \lvert r\rvert$	0.0062	0.0000	0.0000	0.0000	0.0000
$rv \nleftarrow hl$	0.0000	0.0000	0.0000	0.0000	0.0000
$hl \nleftarrow \lvert r\rvert$	0.0000	-	-	-	-
$hl \nleftarrow rv$	0.0000	0.0000	0.0000	0.0000	0.0000
$\lvert r\rvert \nleftarrow rv, hl$	0.0000	0.0000	0.0000	0.0000	0.0000
$rv \nleftarrow \lvert r\rvert, hl$	0.0000	0.0000	0.0000	0.0000	0.0000
$hl \nleftarrow \lvert r\rvert, rv$	0.0000	0.0000	0.0000	0.0000	0.0000

Null hypothesis	CAC40	FTSE100	DAX30	IBEX35	NIKKEI225
$\lvert r\rvert \nleftarrow rv$	0.0000	0.0000	0.0000	0.0000	0.0000
$\lvert r\rvert \nleftarrow hl$	-	-	0.0004	0.0001	0.0000
$rv \nleftarrow \lvert r\rvert$	0.0000	0.0001	0.0000	0.0000	0.0181
$rv \nleftarrow hl$	0.0000	0.0000	0.0000	0.0000	0.0000
$hl \nleftarrow \lvert r\rvert$	0.0464	0.0001	0.0042	0.0001	-
$hl \nleftarrow rv$	0.0000	0.0000	0.0000	0.0000	0.0000
$\lvert r\rvert \nleftarrow rv, hl$	0.0000	0.0000	0.0000	0.0000	0.0000
$rv \nleftarrow \lvert r\rvert, hl$	0.0000	0.0000	0.0000	0.0000	0.0000
$hl \nleftarrow \lvert r\rvert, rv$	0.0000	0.0000	0.0000	0.0000	0.0000

References

Shephard, N. and Sheppard, K. (2010). Realising the future: forecasting with high frequency based volatility (heavy) models. *Journal of Applied Econometrics*, **25**, 197–231.