

APPENDIX OF “ON MARKOV ERROR-CORRECTION MODELS, WITH AN APPLICATION TO STOCK PRICES AND DIVIDENDS”

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In this Appendix, we use Monte Carlo experiments to investigate the usefulness (in finite samples) of the procedures proposed in Section 3.3 of the paper to detect Markov error-correction (MEC) adjustment. These include tests for cointegration, parameter instability, neglected non-linearity and Markov regime switching, as well as a model selection procedure based on the Akaike information criterion (AIC).

1. EXPERIMENTAL DESIGN AND SIMULATION

The data-generating mechanism used in our simulation experiments is the bivariate system

$$y_t + \alpha x_t = z_t, \quad z_t = \{\phi_0 + (\phi_1 - \phi_0)s_t\}z_{t-1} + \varepsilon_{1t}, \quad (\text{A.1})$$

$$y_t + \beta x_t = u_t, \quad u_t = u_{t-1} + \varepsilon_{2t}, \quad (\text{A.2})$$

$$\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \sim \text{NID} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right), \quad (\text{A.3})$$

where $\{s_t\}$ is an ergodic Markov chain on $\{0, 1\}$, independent of $\{(\varepsilon_{1t}, \varepsilon_{2t})\}$, with transition probabilities $p_{00} = \Pr\{s_t = 0 | s_{t-1} = 0\}$ and $p_{11} = \Pr\{s_t = 1 | s_{t-1} = 1\}$. The experiments are a full factorial design of:

$$\alpha = -2, \quad \beta \in \{-3, 0\}, \quad \phi_0 \in \{0, 0.7\}, \quad \phi_1 = 1, \quad \rho \in \{0, -0.5\},$$

$$(p_{00}, p_{11}) \in \{(0.9, 0.9), (0.98, 0.98), (0.98, 0.9), (0.9, 0.98), (0.5, 0.5)\}.$$

The first two pairs of transition probabilities, $(0.9, 0.9)$ and $(0.98, 0.98)$, allow for symmetry in the persistence of the two regimes, the expected duration of each regime being much longer when $p_{00} = p_{11} = 0.98$. The probabilities $(p_{00}, p_{11}) = (0.98, 0.9)$, on the other hand, imply that the regime that corresponds to $s_t = 1$ is considerably less persistent than the regime that corresponds to $s_t = 0$; the opposite is true when $(p_{00}, p_{11}) = (0.9, 0.98)$. Finally, the regime indicators $\{s_t\}$ are uncorrelated when $(p_{00}, p_{11}) = (0.5, 0.5)$. Notice that setting $\beta = 0$ implies that y_t does not react to deviations from long-run equilibrium since an error-correction mechanism is only present in the equation for x_t .

In all the experiments, $50+T$ pseudo-data points for (y_t, x_t) are generated according to (A.1)–(A.3), with $T \in \{50, 100, 200, 500\}$, by setting $z_0 = u_0 = 0$. However, in order to attenuate the effect of the initial values, only the last T pseudo-data points are used for estimation and testing purposes. Unless otherwise stated, the number of Monte Carlo replications per experiment is 2,500.

2. COINTEGRATION TESTS

In our analysis below, we focus on the residual-based ADF , \widehat{Z}_α and \widehat{Z}_t cointegration tests discussed in Phillips and Ouliaris (1990). In our implementation of the tests, the order of the $AR(k)$ model on which the ADF test is based is selected by minimizing the AIC over $k \in \{1, 2, \dots, \lfloor 4(T/100)^{1/4} \rfloor + 1\}$, where $\lfloor \cdot \rfloor$ denotes the greatest-integer function. The \widehat{Z}_α and \widehat{Z}_t tests are used in conjunction with a prewhitened kernel estimator for the long-run innovation variance based on the Parzen kernel function and an automatic plug-in bandwidth (see Andrews and Monahan, 1992). For all three tests, a constant term is included in the test regressions.

We also consider cointegration tests which are based on vector autoregressive models for $\{(y_t, x_t)\}$. These include Stock and Watson's (1988) minimum-eigenvalue test based on their statistic $q_c^\mu(2, 1)$ (denoted by SW) and Johansen's (1991) trace and maximal-eigenvalue likelihood ratio (LR) tests (denoted by LR_{trace} and LR_{max} , respectively). For the latter, the order k (say) of the vector autoregressive model used is determined by minimizing the AIC over $k \in \{1, 2, \dots, \lfloor 4(T/100)^{1/4} \rfloor + 1\}$. The SW test is based on a corrected first-order sample autocorrelation matrix for $(y_t, x_t)'$, where the correction term is estimated nonparametrically using a prewhitened kernel estimator, the Parzen kernel function and an automatic plug-in bandwidth (Andrews and Monahan, 1992). For both types of tests, a constant is included unrestrictedly in the vector autoregressive equations.

Table 1A gives Monte Carlo estimates of the Type-I error probability of 0.05-level cointegration tests in the case where $\beta = 0$ and $\rho = 0$ (the results for $\beta = -3$ are very similar and are therefore omitted in order to save space).¹ Most of the tests have empirical rejection probabilities that are close to the correct rate, the exception being the Johansen tests. The latter tend to be somewhat liberal when the sample size is small, and this must be borne in mind when interpreting results obtained under cointegration.

The empirical rejection probabilities of the tests when cointegration with MEC holds are reported in Table 1B.² It is evident that, although the equilibrium error follows a nonstationary path occasionally, the tests are generally quite powerful to detect the presence of cointegration. This is especially true when the state indicators $\{s_t\}$ are not correlated, a finding which is not perhaps surprising since the frequent state transitions that take place when $p_{00} + p_{11} = 1$ tend to make the equilibrium error look very much like white noise with large variance. For data-generating processes with $p_{00} + p_{11} > 1$, a comparison across different values of the transition probabilities reveals that the probability of correctly detecting cointegration is higher: (a) the more persistent the regime where short-run disequilibrium adjustment takes place is relative to the regime where no such adjustment occurs, and (b) the more observations correspond to the error-correcting regime $s_t = 0$ (as is indeed the case when $p_{00} = 0.98$ and $p_{11} = 0.9$). Also, for any given pair of transition probabilities, the power of the tests rises as the strength of

¹The results for tests at the 0.01 and 0.10 level of significance are qualitatively similar and do not affect the conclusions about the relative merits of different tests.

²In order to reflect empirical practice, all power calculations are done using critical values from the asymptotic null distributions of the test statistics.

disequilibrium adjustment in the regime corresponding to $s_t = 0$ increases (or, equivalently, as the autoregressive coefficient ϕ_0 decreases). This makes it difficult for the tests to detect cointegration with MEC when the equilibrium error is relatively persistent and the sample size is small.

Turning to the individual tests, the \widehat{Z}_α and \widehat{Z}_t tests always have higher empirical rejection probabilities than the *ADF* test, presumably because the nonparametric autocorrelation corrections that the former two tests employ are more successful in accounting for Markov dynamics than the autoregressive approximations on which the *ADF* test is based.³ Among system-wide tests, the *SW* test is generally more powerful than the LR_{trace} and LR_{max} tests, although in most cases the power differences are not very substantial.

Repeating the experiments with $\rho = -0.5$ revealed that the correlation between the innovations of the equilibrium error and the stochastic trend does not contribute to any significant changes in the power of the tests in large samples. For small and moderately-sized samples, the *ADF*, \widehat{Z}_α and \widehat{Z}_t tests suffer a small decrease in power relative to the case with $\rho = 0$, while the opposite is true for the LR_{trace} and LR_{max} tests.

In summary, our analysis shows that conventional cointegration tests based on the assumption of a linear adjustment process are capable of detecting the presence of a equilibrium relationship between MEC time series.

3. TESTS FOR PARAMETER INSTABILITY

The tests for parameter stability we consider are based on the following statistics: (a) Nyblom's (1989) *L* statistic (modified as in Hansen (1992a) to achieve robustness with respect to heteroskedasticity), which provides a locally most powerful test for the null hypothesis of parameter stability against the alternative of martingale parameter variation; (b) functionals of the sequence of LR statistics which test the null hypothesis of parameter stability against the alternative of an one-time break at all possible break-points in the sample; following Andrews (1993) and Andrews and Ploberger (1994), the statistics considered are

$$\begin{aligned} AvgLR &= \int_{\omega_1}^{\omega_2} LR(\omega) d\omega, & ExpLR &= \ln \left\{ \int_{\omega_1}^{\omega_2} \exp\left[\frac{1}{2}LR(\omega)\right] d\omega \right\}, \\ SupLR &= \sup_{\omega \in (\omega_1, \omega_2)} LR(\omega), \end{aligned}$$

where $LR(\omega)$ denotes the LR statistic for testing for a change at date ωT , $\omega \in (0, 1)$. The tests are implemented with $\omega_0 = 1 - \omega_1 = 0.15$.

For computational convenience, we set $\beta = 0$ in the data-generating process so that only x_t responds to deviations from the long-run equilibrium relationship. Further, in order to reflect empirical practice, we treat the cointegrating vector as unknown and estimate it from the data. Thus, in each Monte Carlo replication, we proceed according to the following two-step procedure.

³These findings are similar to the results of van Dijk and Franses (1996) and Balke and Fomby (1997), who found the \widehat{Z}_α and \widehat{Z}_t tests to be more powerful than the *ADF* test in the presence of threshold cointegration.

First, we obtain an estimate $(1, \hat{\alpha})$ of the cointegrating vector $(1, \alpha)$ by means of the fully-modified least squares method of Phillips and Hansen (1990); long-run covariance matrices are estimated as in Andrews and Monahan (1992), using a prewhitened kernel estimator, the Parzen kernel function and a data-dependent automatic bandwidth. Second, we test for parameter instability in: (a) an AR(1) model for $\hat{z}_t = y_t + \hat{\alpha}x_t$:

$$\text{Model M}_1: \quad \hat{z}_t = b_0 + b_1 \hat{z}_{t-1} + v_{1t}^*$$

(b) an error-correction model for x_t :

$$\text{Model M}_2: \quad \Delta x_t = c_0 + c_1 \hat{z}_{t-1} + v_{2t}^*.$$

The top panel in Tables 2A–2B gives Monte Carlo estimates of the empirical rejection probabilities of 0.05-level tests for models M_1 and M_2 when $\rho = 0$ and the null hypothesis of parameter constancy is true (i.e. $\phi_1 = \phi_0$, $|\phi_0| < 1$).⁴ Tests in the error-correction model M_2 generally have rejection probabilities that are not very different from the 0.05 nominal value. This is not the case in model M_1 where all tests tend to be somewhat liberal even for relatively large sample sizes.

Turning to the rejection probabilities of the tests in the presence of MEC adjustment, also shown in Tables 2A–2B, it is obvious that the performance of the tests is disappointing when $p_{00} = p_{11} = 0.5$. In this case, there are frequent transitions between the two regimes, and the power of the tests suffers as a result. Fortunately, however, matters improve considerably when $p_{00} + p_{11} > 1$. In model M_1 , tests other than L are capable of detecting parameter non-constancy, especially when the change in the autoregressive parameter is relatively large (i.e., $\phi_0 = 0$). The rejection probabilities of the tests are lower in the error-correction model M_2 , presumably due to the fact that the changes in the coefficient of the error-correction mechanism that are implied by our data-generating process are small (this coefficient switches between 0 and 0.5 when $\phi_0 = 0$ and between 0 and 0.15 when $\phi_0 = 0.7$). Finally, the *ExpLR* and *SupLR* tests are consistently more successful than the *AvgLR* and L tests. The latter is very weak compared to the other tests, which is not perhaps surprising since the test is designed for situations where there is a relatively constant likelihood of parameter variation throughout the sample.

4. TESTS FOR NEGLECTED NONLINEARITY

Further useful information about the adequacy or otherwise of a linear adjustment process may be obtained from application of tests for neglected nonlinearity in the relevant error-correction model or in an autoregressive model for the equilibrium error. We now investigate the properties of such tests in the context of the linear models M_1 (AR(1) model for \hat{z}_t) and M_2 (error-correction model for x_t) considered in the experiments of the previous section. Each of these models is tested for neglected nonlinearity using the following tests: (i) the modified regression equation specification error test (*RESET*) of Thursby and Schmidt (1977), with powers of regressors up to 4; (ii) the *BDS* test of Brock et al. (1996), with embedding dimension

⁴Results for $\rho = -0.5$ are very similar to those obtained with $\rho = 0$, and are not, therefore, reported.

equal to 2 and metric bound equal to the standard deviation of the estimated residuals; (iii) the ‘WHITE3’ dynamic information matrix test discussed in Lee et al. (1993) (denoted by *WHT*); (iv) the ‘NEURAL2’ neural network test of Lee et al. (1993), based on a logistic squashing function and the second and third largest principal components of 20 randomly generated unit signals (denoted by *NNT*).

Tables 3A–3B report the empirical rejection probabilities of 0.05-level nonlinearity tests for data-generating processes identical to those considered in subsection 3.2, with $\beta = \rho = 0$.⁵ With the exception of the *BDS* test, nonlinearity tests reject at the correct rate when the null hypothesis of linearity is true ($\phi_1 = \phi_0$), even for the smaller sample sizes. In the presence of Markov cointegration, tests for neglected nonlinearity in model M_1 perform poorly when $p_{00} = p_{11} = 0.5$, with only the *BDS* test being capable of rejecting the linear model (although the rejection probabilities for the two smaller sample sizes is misleading since the test tends to over-reject under the null). Allowing the chain $\{s_t\}$ to be fairly persistent leads to a considerable improvement in the performance of the tests, especially when $T \geq 200$. In these cases, the *RESET* and *WHT* tests perform best overall but test power remains low for small differences between ϕ_1 and ϕ_0 .

The overall picture is the same for tests applied to model M_2 . Here, however, test rejection frequencies are generally lower than those obtained for model M_1 , presumably because Markov nonlinearity is less prominent in the error-correction model as a result of the shifts in coefficients being relatively small.

5. A TEST FOR MARKOV SWITCHING

The previous two sections have demonstrated that tests for parameter instability and neglected nonlinearity can provide useful insights into the validity or otherwise of the assumption of a linear adjustment process. It should be appreciated, however, that such tests are not designed to test departures from linearity or stability in the direction of Markov switching models, and they are reasonably powerful against several types of nonlinearity and parameter nonconstancy. Thus, in the absence of additional information, rejection of the hypothesis of stability or linearity on the basis of these tests cannot be necessarily accepted as evidence in favour of Markov switching behaviour.

In the face of these difficulties, it is clearly desirable to complement parameter instability and nonlinearity tests with procedures which directly test the hypothesis of linear adjustment towards equilibrium against a Markov alternative. Unfortunately, this testing problem is non-standard in that the transition probabilities are unidentified and scores are identically zero under the null hypothesis of linearity, thus violating conventional regularity conditions for likelihood-based inference. Hansen (1992b) proposed a general theory for testing under such non-standard conditions. By viewing the likelihood function as an empirical process of the unknown parameters, a bound for the asymptotic distribution of a suitably standardized LR statistic can be obtained. This asymptotic distribution is generally non-standard, but an approximation to it

⁵Very similar results were obtained with $\rho = -0.5$.

may be obtained via simulation. The difficulty with this method is that it involves evaluation of the likelihood function across a grid of different values for the transition probabilities and for each state-dependent parameter and hence is extremely intensive computationally.

The data-generating process for the simulations is similar to that used in the previous two sections, although, due to the exceptionally high computational cost of the experiments, only the following parameter values and sample sizes are considered:

$$\alpha = -2, \quad \beta = 0, \quad \phi_0 \in \{0, 0.7\}, \quad \phi_1 = 1, \quad \rho = 0,$$

$$(p_{00}, p_{11}) \in \{(0.5, 0.5), (0.9, 0.9), (0.98, 0.9)\}, \quad T \in \{50, 100, 200\}.$$

For each design point, we test model M_1 (single-state AR(1) model for \hat{z}_t) and model M_2 (single-state error-correction model for x_t) against corresponding Markov alternatives (i.e. two-state Markov switching models), using Hansen's (1992b) standardized LR statistic. In both cases, a constant is also included in the models but, in order to reduce the computation times, it is assumed to be state-independent. For the calculations, we use for grid for the state-dependent coefficients the range $[0.01, 1.01]$ in steps of 0.1 (11 gridpoints), while the range $[0.50, 0.95]$ in steps of 0.05 (10 gridpoints) is used for the transition probabilities. The asymptotic p-values of the tests are calculated according to the method described in Hansen (1996), using 1,000 random draws from the relevant limiting Gaussian processes and bandwidth parameter $M \in \{0, 1, \dots, 4\}$.

Table 4 records the empirical rejection probabilities of the tests (calculated as the fraction of 500 Monte Carlo trials in which the test p-value did not exceed 0.05). It is clear that, when testing the AR(1) model for \hat{z}_t , the likelihood ratio test is fairly powerful if the Markov chain is persistent and $\phi_0 = 0$. However, the test has virtually no power to detect Markov switching behaviour when the difference between ϕ_1 and ϕ_0 is small (the problem is, of course, exacerbated by the fact that our test procedure uses asymptotic p-values which are only an upper bound for the true p-values and hence the test tends to be conservative). A similar picture emerges when testing the error-correction model for x_t , in which case the rejection probabilities are lower than those obtained for model M_1 (due to the fact that the changes in the coefficient of the error-correction mechanism that are implied by our data-generating process are small). Finally, as with parameter instability tests, the Hansen test is more successful in detecting Markov switching the more autocorrelated the hidden regime indicators are.

6. A MODEL SELECTION APPROACH

An alternative way of distinguishing between cointegration models with linear adjustment and cointegration models with MEC adjustment is by comparing the rival models on the basis of a complexity-penalized likelihood criterion (e.g., the AIC). As Granger et al. (1995) point out, such a method is arguably more appropriate for model selection than procedures based on formal hypothesis testing, partly because, unlike testing, it does not favour unfairly the model chosen to be the null hypothesis. This last point is particularly important in the case of MEC adjustment since all the procedures considered so far have a linear cointegration model as a null

hypothesis. To make matters worse, one of the tests (namely Hansen’s test) is conservative by construction, thus further favouring linear models.

Here, we investigate the finite-sample performance of a selection procedure based on the popular AIC under model (A.1)–(A.3) with $\beta = 0$. In each Monte Carlo replication, we calculate the value of the AIC for the linear models M_1 and M_2 (which include a constant) and the corresponding Markov models (with switching intercept and slope).

The empirical probabilities of correctly selecting the Markov two-state models instead of the corresponding single-state specifications are reported in Tables 5A–5B. In the case of autoregressive models for \hat{z}_t , the AIC performs extremely well when $\phi_0 = 0$ and $T \geq 100$. For the error-correction model for Δx_t , roughly the same picture emerges, although the AIC is somewhat less successful in selecting the right model, especially when $p_{00} + p_{11} = 1$. As with all previous procedures, difficulties are encountered when $\phi_0 = 0.7$, but even in these cases one is more likely to arrive at the correct conclusion about the presence of MEC adjustment using the AIC than the nonlinearity tests of the type discussed in the last two subsections. There appear, therefore, to be good reasons for using a model selection procedure as a further means of establishing the presence or otherwise of Markov-type nonlinearity in the adjustment process.

7. CONCLUSION

On the basis of our simulation results, we recommend starting with the \hat{Z}_α , \hat{Z}_t and *SW* cointegration tests, which have high power to detect the presence of a long-run relationship among cointegrated MEC time series. In the second step, the *ExpLR* and *SupLR* tests for parameter instability can be useful in revealing the invalidity of the assumption of a continuous and constant-strength adjustment process. These could be supplemented with the *RESET* and *WHT* tests for neglected nonlinearity both of which have respectable power against Markov-type alternatives. Better still, if the high computational cost is acceptable, Hansen’s (1992b) procedure can be used to directly test the one-state linear model of interest against the corresponding Markov model. Of course, since there exist situations in which all these tests tend to have low power, it seems prudent to anyhow fit both linear and Markov switching models to the data. Then, one of the competing models may be selected by using the AIC criterion, which was found to work well in our simulations.

Table 1A. Rejection Probabilities (%) of Cointegration Tests
 $(\phi_0 = \phi_1 = 1)$

T	ADF	\widehat{Z}_α	\widehat{Z}_t	SW	LR_{trace}	LR_{max}
50	4.84	6.44	7.80	5.04	9.80	9.76
100	4.28	6.64	6.16	4.88	7.32	7.36
200	4.76	6.48	5.96	5.68	7.20	7.72
500	5.40	5.48	5.20	5.08	6.28	5.72

Table 1B. Rejection Probabilities (%) of Cointegration Tests $(\phi_1 = 1)$

	$T = 50$		$T = 100$		$T = 200$		$T = 500$	
	ϕ_0							
			$p_{00} = 0.5, p_{11} = 0.5$					
ADF		44.60 8.68	92.36 25.24	99.76 81.40	100.0 100.0	100.0 100.0	100.0 100.0	
\widehat{Z}_α		75.60 15.16	99.32 42.64	100.0 93.00	100.0 100.0	100.0 100.0	100.0 100.0	
\widehat{Z}_t		78.60 16.08	99.12 35.84	100.0 89.56	100.0 100.0	100.0 100.0	100.0 100.0	
SW		83.44 12.32	99.64 40.40	100.0 93.24	100.0 100.0	100.0 100.0	100.0 100.0	
LR_{trace}		56.24 16.72	94.72 30.00	99.96 74.68	100.0 100.0	100.0 100.0	100.0 100.0	
LR_{max}		56.92 14.64	95.76 27.60	99.96 77.80	100.0 100.0	100.0 100.0	100.0 100.0	
			$p_{00} = 0.9, p_{11} = 0.9$					
ADF		16.40 7.36	33.52 15.24	60.36 40.76	97.64 92.56	97.64 92.56	97.64 92.56	
\widehat{Z}_α		34.24 11.48	56.28 26.20	79.36 50.76	99.24 95.52	99.24 95.52	99.24 95.52	
\widehat{Z}_t		38.00 12.32	55.16 21.92	78.00 46.36	99.00 93.64	99.00 93.64	99.00 93.64	
SW		37.68 9.56	58.68 23.44	83.60 52.16	99.64 96.80	99.64 96.80	99.64 96.80	
LR_{trace}		23.08 14.00	39.28 21.00	65.80 41.04	97.72 89.28	97.72 89.28	97.72 89.28	
LR_{max}		21.48 12.20	38.56 18.68	66.20 39.88	98.48 90.64	98.48 90.64	98.48 90.64	
			$p_{00} = 0.98, p_{11} = 0.98$					
ADF		13.24 7.64	15.76 12.20	20.32 21.20	36.48 39.32	36.48 39.32	36.48 39.32	
\widehat{Z}_α		27.40 11.32	33.28 20.16	37.92 27.72	56.08 46.04	56.08 46.04	56.08 46.04	
\widehat{Z}_t		30.36 12.24	32.80 16.96	37.64 25.28	53.68 43.04	53.68 43.04	53.68 43.04	
SW		28.60 9.32	34.28 18.68	41.40 28.60	59.48 48.08	59.48 48.08	59.48 48.08	
LR_{trace}		17.40 12.52	21.84 15.48	26.80 22.84	42.28 37.36	42.28 37.36	42.28 37.36	
LR_{max}		17.64 12.12	21.00 14.72	25.32 21.84	42.68 37.12	42.68 37.12	42.68 37.12	

Table 1C. Rejection Probabilities (%) of Cointegration Tests ($\phi_1 = 1$)

	$T = 50$		$T = 100$		$T = 200$		$T = 500$	
	ϕ_0							
			$p_{00} = 0.98, p_{11} = 0.9$					
<i>ADF</i>								
\widehat{Z}_α								
\widehat{Z}_t								
<i>SW</i>								
LR_{trace}								
LR_{max}								
			$p_{00} = 0.9, p_{11} = 0.98$					
<i>ADF</i>								
\widehat{Z}_α								
\widehat{Z}_t								
<i>SW</i>								
LR_{trace}								
LR_{max}								

Table 2A. Rejection Probabilities (%) of Parameter Instability Tests: Model M_1

ϕ_0	$T = 50$		$T = 100$		$T = 200$		$T = 500$	
	0.0	0.7	0.0	0.7	0.0	0.7	0.0	0.7
$\phi_1 = \phi_0$								
<i>L</i>	6.32	2.24	7.64	3.64	9.72	4.64	12.32	6.40
<i>AvgLR</i>	7.84	8.40	8.88	8.60	10.64	7.60	12.80	8.80
<i>ExpLR</i>	8.96	11.76	9.76	9.80	11.48	8.32	12.76	8.32
<i>SupLR</i>	5.92	9.04	7.32	8.92	9.20	7.72	11.40	7.76
$\phi_1 = 1, p_{00} = 0.5, p_{11} = 0.5$								
<i>L</i>	2.60	2.36	4.40	1.80	5.12	2.68	7.20	5.44
<i>AvgLR</i>	7.72	12.84	12.24	11.48	15.16	10.08	19.04	10.64
<i>ExpLR</i>	14.48	17.76	15.88	14.52	17.56	11.52	22.24	10.76
<i>SupLR</i>	13.56	16.08	17.04	14.84	18.24	12.80	24.60	11.08
$\phi_1 = 1, p_{00} = 0.9, p_{11} = 0.9$								
<i>L</i>	8.12	2.64	10.60	2.80	15.00	3.68	21.48	7.68
<i>AvgLR</i>	30.12	17.92	40.04	16.64	50.60	17.28	61.32	23.48
<i>ExpLR</i>	45.44	26.20	56.72	23.68	65.60	25.44	75.92	30.80
<i>SupLR</i>	40.84	23.08	55.28	24.28	67.96	29.16	79.64	37.48
$\phi_1 = 1, p_{00} = 0.98, p_{11} = 0.98$								
<i>L</i>	13.00	3.16	21.96	5.12	26.84	7.60	28.76	12.44
<i>AvgLR</i>	47.00	23.72	63.16	30.04	68.60	38.20	70.48	46.16
<i>ExpLR</i>	58.68	33.28	72.68	39.32	79.12	48.96	82.92	59.76
<i>SupLR</i>	53.04	29.08	70.20	37.20	79.20	49.76	84.76	65.00
$\phi_1 = 1, p_{00} = 0.98, p_{11} = 0.9$								
<i>L</i>	14.12	2.96	25.68	3.68	39.64	8.72	58.68	23.36
<i>AvgLR</i>	41.76	19.52	66.12	22.04	80.32	31.88	92.28	48.28
<i>ExpLR</i>	54.72	26.96	75.24	27.92	87.56	35.76	96.36	56.16
<i>SupLR</i>	49.08	24.04	72.52	27.64	88.16	34.72	97.00	57.72
$\phi_1 = 1, p_{00} = 0.9, p_{11} = 0.98$								
<i>L</i>	7.64	3.56	6.96	3.44	6.88	2.80	5.40	2.16
<i>AvgLR</i>	33.64	23.96	38.28	24.80	37.40	23.44	35.08	19.32
<i>ExpLR</i>	51.28	35.40	56.04	36.72	55.96	37.24	56.04	35.64
<i>SupLR</i>	46.56	31.92	55.44	37.76	58.64	42.64	62.04	44.08

Table 2B. Rejection Probabilities (%) of Parameter Instability Tests: Model M_2

	$T = 50$		$T = 100$		$T = 200$		$T = 500$		
	ϕ_0	0.0	0.7	0.0	0.7	0.0	0.7	0.0	0.7
	$\phi_1 = \phi_0$								
<i>L</i>	4.16	2.64	5.56	3.92	6.52	3.80	6.76	4.88	
<i>AvgLR</i>	6.44	6.48	6.68	6.16	6.60	5.40	6.64	5.28	
<i>ExpLR</i>	7.48	7.72	6.64	6.40	6.08	5.96	6.08	5.04	
<i>SupLR</i>	5.04	5.56	4.68	5.12	4.60	5.12	5.12	4.56	
	$\phi_1 = 1, p_{00} = 0.5, p_{11} = 0.5$								
<i>L</i>	2.00	2.72	3.28	2.88	3.36	2.76	4.16	4.32	
<i>AvgLR</i>	5.92	7.80	7.08	6.64	8.80	6.28	10.56	6.28	
<i>ExpLR</i>	8.16	9.32	8.84	7.32	10.12	6.92	12.00	6.36	
<i>SupLR</i>	7.96	7.64	8.48	6.60	10.08	6.96	13.48	6.08	
	$\phi_1 = 1, p_{00} = 0.9, p_{11} = 0.9$								
<i>L</i>	3.76	2.80	6.56	2.80	8.48	3.08	12.44	4.76	
<i>AvgLR</i>	15.96	10.20	21.60	8.60	29.52	8.48	41.00	11.56	
<i>ExpLR</i>	26.40	13.80	34.44	11.00	42.88	11.08	55.56	14.00	
<i>SupLR</i>	23.32	11.40	33.32	10.28	45.44	12.88	59.96	17.32	
	$\phi_1 = 1, p_{00} = 0.98, p_{11} = 0.98$								
<i>L</i>	7.52	3.36	11.96	3.76	14.92	3.60	17.68	6.16	
<i>AvgLR</i>	25.28	13.76	38.00	12.64	43.52	15.24	50.60	21.32	
<i>ExpLR</i>	37.44	19.28	49.80	18.64	56.96	21.52	65.32	30.64	
<i>SupLR</i>	34.44	15.60	48.08	18.04	58.16	22.68	69.16	35.52	
	$\phi_1 = 1, p_{00} = 0.98, p_{11} = 0.9$								
<i>L</i>	6.96	2.80	14.52	3.04	23.60	4.48	42.64	11.92	
<i>AvgLR</i>	23.24	9.52	39.64	10.64	59.28	13.04	79.56	25.48	
<i>ExpLR</i>	34.08	13.40	49.92	13.24	68.44	15.68	88.00	28.64	
<i>SupLR</i>	29.12	11.36	46.40	12.16	69.08	15.68	89.72	29.08	
	$\phi_1 = 1, p_{00} = 0.9, p_{11} = 0.98$								
<i>L</i>	3.88	3.72	3.80	2.48	3.08	2.64	2.96	2.40	
<i>AvgLR</i>	16.84	12.68	19.16	12.44	18.48	10.40	15.92	7.52	
<i>ExpLR</i>	31.60	20.08	34.88	18.84	35.72	17.16	31.84	13.72	
<i>SupLR</i>	29.24	17.76	35.00	18.88	38.48	20.00	39.80	20.52	

Table 3A. Rejection Probabilities (%) of Nonlinearity Tests: Model M₁

ϕ_0	$T = 50$		$T = 100$		$T = 200$		$T = 500$	
	0.0	0.7	0.0	0.7	0.0	0.7	0.0	0.7
	$\phi_1 = \phi_0$							
<i>RESET</i>	4.44	3.28	3.80	3.92	4.48	3.28	4.76	3.24
<i>WHT</i>	3.84	4.80	4.48	4.80	4.08	3.20	5.00	4.60
<i>BDS</i>	18.20	17.48	10.84	10.20	7.64	7.76	5.80	5.52
<i>NNT</i>	4.16	4.08	3.68	4.20	4.32	4.52	4.40	4.72
	$\phi_1 = 1, p_{00} = 0.5, p_{11} = 0.5$							
<i>RESET</i>	18.24	5.40	26.20	5.92	32.16	7.60	43.48	8.56
<i>WHT</i>	13.56	6.44	20.04	6.52	23.56	6.76	31.00	7.76
<i>BDS</i>	33.56	16.60	50.32	10.96	79.44	8.76	99.24	10.52
<i>NNT</i>	11.08	3.72	13.64	4.56	16.12	5.20	20.96	6.12
	$\phi_1 = 1, p_{00} = 0.9, p_{11} = 0.9$							
<i>RESET</i>	23.56	5.60	37.76	7.20	53.56	10.08	75.96	20.44
<i>WHT</i>	16.52	6.12	31.04	7.20	57.48	10.84	92.52	24.72
<i>BDS</i>	26.20	18.28	33.28	11.44	53.76	9.76	87.40	10.00
<i>NNT</i>	11.20	3.96	19.96	4.40	40.32	6.28	81.00	25.64
	$\phi_1 = 1, p_{00} = 0.98, p_{11} = 0.98$							
<i>RESET</i>	24.48	7.92	41.96	11.24	54.68	18.92	68.60	35.60
<i>WHT</i>	20.32	6.88	41.52	7.84	69.36	14.12	91.92	35.68
<i>BDS</i>	25.36	18.88	30.20	11.80	49.16	11.28	82.48	12.76
<i>NNT</i>	12.20	5.04	24.88	6.72	42.28	11.96	63.12	34.40
	$\phi_1 = 1, p_{00} = 0.98, p_{11} = 0.9$							
<i>RESET</i>	24.64	5.52	48.92	9.24	77.12	19.08	97.40	50.12
<i>WHT</i>	19.56	6.00	50.48	7.12	86.88	13.08	99.92	36.36
<i>BDS</i>	24.64	17.96	36.60	11.52	69.72	8.92	99.08	9.68
<i>NNT</i>	12.84	4.12	29.48	5.88	63.88	14.08	97.04	55.16
	$\phi_1 = 1, p_{00} = 0.9, p_{11} = 0.98$							
<i>RESET</i>	21.32	6.92	30.40	9.16	36.44	10.04	43.08	11.56
<i>WHT</i>	16.96	8.36	24.04	9.32	34.72	14.72	57.68	31.00
<i>BDS</i>	27.00	19.68	25.96	13.28	31.40	12.12	49.68	14.84
<i>NNT</i>	9.80	3.88	15.64	5.40	19.52	5.24	29.20	7.20

Table 3B. Rejection Probabilities (%) of Nonlinearity Tests: Model M_2

	$T = 50$		$T = 100$		$T = 200$		$T = 500$		
	ϕ_0	0.0	0.7	0.0	0.7	0.0	0.7	0.0	0.7
	$\phi_1 = \phi_0$								
<i>RESET</i>		4.20	3.56	4.04	4.48	4.28	4.92	4.20	3.76
<i>WHT</i>		5.16	4.12	4.68	3.44	5.20	4.96	5.08	4.56
<i>BDS</i>		19.92	17.68	10.48	11.60	7.84	7.76	5.92	5.84
<i>NNT</i>		4.00	4.24	3.80	4.96	4.76	5.20	4.40	4.36
	$\phi_1 = 1, p_{00} = 0.5, p_{11} = 0.5$								
<i>RESET</i>		12.28	5.08	17.36	5.52	21.36	6.20	29.16	7.84
<i>WHT</i>		9.60	4.08	11.92	4.20	13.96	5.52	19.00	5.76
<i>BDS</i>		19.84	17.40	16.04	12.00	23.16	7.80	45.80	6.36
<i>NNT</i>		9.36	4.16	10.68	4.68	11.48	5.08	15.16	5.40
	$\phi_1 = 1, p_{00} = 0.9, p_{11} = 0.9$								
<i>RESET</i>		17.00	6.40	26.28	7.00	40.16	7.44	61.12	12.88
<i>WHT</i>		11.36	4.56	17.88	4.76	32.72	7.24	66.52	12.16
<i>BDS</i>		20.76	18.44	15.84	11.20	19.28	8.32	30.20	6.48
<i>NNT</i>		8.72	5.12	12.20	4.68	26.72	5.04	69.08	14.24
	$\phi_1 = 1, p_{00} = 0.98, p_{11} = 0.98$								
<i>RESET</i>		19.00	6.48	29.68	8.12	42.20	11.40	58.84	21.40
<i>WHT</i>		13.60	6.24	24.36	6.92	50.40	10.56	84.08	19.60
<i>BDS</i>		22.52	18.56	18.68	12.64	23.68	10.44	40.68	9.04
<i>NNT</i>		8.48	4.96	16.68	5.44	29.96	7.96	52.28	20.32
	$\phi_1 = 1, p_{00} = 0.98, p_{11} = 0.9$								
<i>RESET</i>		18.60	5.56	35.20	6.92	61.00	10.28	93.68	30.04
<i>WHT</i>		12.92	4.48	24.96	4.76	56.56	7.44	94.36	13.92
<i>BDS</i>		19.84	18.00	17.20	11.00	23.48	8.48	51.16	6.96
<i>NNT</i>		9.28	3.88	19.16	5.24	48.40	8.04	93.28	32.96
	$\phi_1 = 1, p_{00} = 0.9, p_{11} = 0.98$								
<i>RESET</i>		18.08	7.96	23.48	8.60	29.12	8.20	34.92	9.80
<i>WHT</i>		12.28	6.68	18.56	8.24	25.52	12.16	39.24	19.64
<i>BDS</i>		22.32	18.32	19.60	13.88	16.32	10.84	19.60	10.28
<i>NNT</i>		8.00	4.24	9.28	4.56	12.24	4.56	19.40	6.68

Table 4A. Rejection Probabilities (%) of Hansen's Test: Model for \hat{z}_t ($\phi_1 = 1$)

ϕ_0	$T = 50$		$T = 100$		$T = 200$	
	0.0	0.7	0.0	0.7	0.0	0.7
	$p_{00} = 0.5, \quad p_{11} = 0.5$					
$M = 0$	11.20	0.40	26.00	0.80	69.40	1.20
$M = 1$	11.40	0.60	26.80	0.80	69.20	1.60
$M = 2$	12.60	0.80	28.40	0.80	69.40	1.40
$M = 3$	14.20	1.20	29.40	1.00	69.20	1.20
$M = 4$	14.60	1.20	29.60	1.00	69.20	1.40
	$p_{00} = 0.9, \quad p_{11} = 0.9$					
$M = 0$	16.40	1.80	50.20	2.00	82.80	6.60
$M = 1$	16.00	1.40	49.20	1.80	82.40	5.00
$M = 2$	16.20	1.20	47.00	1.40	80.60	3.80
$M = 3$	15.80	1.40	45.20	1.20	80.00	3.20
$M = 4$	15.80	1.00	45.00	1.20	79.40	3.00
	$p_{00} = 0.98, \quad p_{11} = 0.9$					
$M = 0$	19.40	0.60	52.00	2.40	87.80	10.60
$M = 1$	18.40	0.60	50.20	2.00	86.40	9.40
$M = 2$	17.60	0.60	46.20	1.80	85.20	8.20
$M = 3$	17.00	0.60	44.60	1.80	85.00	7.60
$M = 4$	16.60	0.60	42.80	1.80	83.80	7.00

Table 4B. Rejection Probabilities (%) of Hansen's Test: Model for Δx_t ($\phi_1 = 1$)

ϕ_0	$T = 50$		$T = 100$		$T = 200$	
	0.0	0.7	0.0	0.7	0.0	0.7
	$p_{00} = 0.5, \quad p_{11} = 0.5$					
$M = 0$	5.20	1.00	6.60	0.40	20.00	0.60
$M = 1$	5.40	1.40	6.60	0.40	21.20	0.70
$M = 2$	5.60	1.40	7.80	0.60	21.20	0.80
$M = 3$	6.00	1.80	8.00	0.60	21.80	0.80
$M = 4$	6.80	1.60	9.20	0.60	22.40	0.60
	$p_{00} = 0.9, \quad p_{11} = 0.9$					
$M = 0$	13.20	1.00	23.40	0.40	45.40	1.20
$M = 1$	13.00	0.80	23.00	0.20	44.40	1.00
$M = 2$	12.60	0.80	22.60	0.20	43.80	0.80
$M = 3$	12.40	1.20	21.60	0.20	43.00	0.60
$M = 4$	12.20	1.20	20.80	0.20	43.20	0.60
	$p_{00} = 0.98, \quad p_{11} = 0.9$					
$M = 0$	10.60	0.60	22.00	0.80	59.80	2.00
$M = 1$	10.40	0.60	20.80	0.80	58.60	2.00
$M = 2$	10.60	0.60	20.20	0.80	57.00	1.40
$M = 3$	11.00	0.60	19.00	0.80	54.80	1.20
$M = 4$	10.80	0.60	18.00	1.00	53.60	1.00

Table 5A. Selection Probabilities (%): Model for \hat{z}_t ($\phi_1 = 1$)

ϕ_0	$T = 50$		$T = 100$		$T = 200$		$T = 500$	
	0.0	0.7	0.0	0.7	0.0	0.7	0.0	0.7
			$p_{00} = 0.5, p_{11} = 0.5$					
	48.68	21.36	69.64	22.80	92.12	26.24	100.0	42.68
			$p_{00} = 0.9, p_{11} = 0.9$					
	57.72	23.68	83.48	28.16	98.00	42.48	100.0	75.28
			$p_{00} = 0.98, p_{11} = 0.98$					
	57.16	22.40	77.96	27.76	89.16	42.36	96.88	74.60
			$p_{00} = 0.98, p_{11} = 0.9$					
	55.88	21.84	83.60	26.84	98.12	43.56	100.0	79.60
			$p_{00} = 0.9, p_{11} = 0.98$					
	58.12	26.04	77.52	32.24	88.32	44.04	97.80	73.04

Table 5B. Selection Probabilities (%): Model for Δx_t ($\phi_1 = 1$)

ϕ_0	$T = 50$		$T = 100$		$T = 200$		$T = 500$	
	0.0	0.7	0.0	0.7	0.0	0.7	0.0	0.7
			$p_{00} = 0.5, p_{11} = 0.5$					
	31.36	16.56	42.84	17.24	65.64	19.08	95.32	23.84
			$p_{00} = 0.9, p_{11} = 0.9$					
	41.64	17.24	61.64	16.52	88.00	23.24	99.64	40.08
			$p_{00} = 0.98, p_{11} = 0.98$					
	45.80	17.24	63.16	19.44	79.44	26.80	91.28	48.88
			$p_{00} = 0.98, p_{11} = 0.9$					
	39.64	15.72	60.60	16.20	89.68	21.84	99.80	45.96
			$p_{00} = 0.9, p_{11} = 0.98$					
	51.32	21.28	66.88	26.04	81.44	32.52	94.72	53.00