Mixed Frequency Structural Models: Identification, Estimation, and Policy Analysis

Online Appendix

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24 July 2013

1 Smets and Wouters (2007, SW): the model equations

We list here the equations which describe the dynamics of the SW model.

The model variables for the sticky wage and price economy are: output (y_t), consumption (c_t), investment (i_t), Tobin's q (q_t), utilized capital (k_t^s), installed capital (k_t), capacity utilization (z_t), rental rate of capital (r_t^k), price markup (μ_t^p), inflation rate (π_t), wage markup (μ_t^w), real wage (w_t), total hours worked (l_t), and nominal interest rate (r_t). For the corresponding flexible economy: output (y_t^*), consumption (c_t^*), investment (i_t^*), Tobin's q (q_t^*), utilized capital (k_t^{s*}), installed capital (k_t^*), capacity utilization (z_t^*), rental rate of capital (r_t^{k*}), price markup (μ_t^{p*}), wage markup (μ_t^{w*}), real wage (w_t^*), and total hours worked (l_t^*), for the corresponding flexible economy.

The shocks are: total factor productivity (ε_t^a), investment-specific technology (ε_t^i), government purchases (ε_t^g), risk premium (ε_t^b), monetary policy (ε_t^r), wage markup (ε_t^w) and price markup (ε_t^p).

<u>Flexible economy:</u>

$$\varepsilon_t^a = \alpha r_t^{k*} + (1-\alpha)\, w_t^* \tag{1}$$

$$z_t^* = \frac{1}{\frac{\psi}{1-\psi}} r_t^{k*} \tag{2}$$

$$k_t^{s*} = z_t^* + k_{t-1}^* \tag{3}$$

$$r_t^{k*} = w_t^* + l_t^* - k_t^{s*} \tag{4}$$

$$y_t^* = c_y c_t^* + i_y i_t^* + r^{kss} k_y z_t^* + \varepsilon_t^g \tag{5}$$

$$y_t^* = \Phi\left(\alpha\, k_t^{s*} + (1-\alpha)\; l_t^* + \varepsilon_t^a\right) \tag{6}$$

$$i_t^* = \frac{1}{1+\beta\,\gamma^{(1-\sigma_c)}}\left(i_{t-1}^* + \beta\,\gamma^{(1-\sigma_c)} E_t\, i_{t+1}^* + \frac{1}{\gamma^2\,\varphi}\, q_t^*\right) + \varepsilon_t^i \tag{7}$$

$$q_t^* = -r_t^* + \left(1-\beta\,(1-\delta)\,\gamma^{-\sigma_c}\right) E_t\, r_{t+1}^{k*} + \beta\,(1-\delta)\,\gamma^{-\sigma_c} E_t q_{t+1}^* - \varepsilon_t^b \tag{8}$$

$$c_t^* = \frac{\frac{h}{\gamma}}{1+\frac{h}{\gamma}}\, c_{t-1}^* + \frac{1}{1+\frac{h}{\gamma}}\, E_t c_{t+1}^* + \frac{(\sigma_c-1)\; w^{ss} l^{ss}}{c^{ss}\sigma_c\left(1+\frac{h}{\gamma}\right)}\,\left(l_t^* - E_t l_{t+1}^*\right) \tag{9}$$

$$- \frac{1-\frac{h}{\gamma}}{\sigma_c\left(1+\frac{h}{\gamma}\right)} r_t^* + \varepsilon_t^b$$

$$w_t^* = \sigma_l l_t^* + \frac{1}{1-\frac{h}{\gamma}} c_t^* - \frac{\frac{h}{\gamma}}{1-\frac{h}{\gamma}} c_{t-1}^* \tag{10}$$

$$k_t^* = \frac{(1-\delta)}{\gamma} k_{t-1}^* + \left(1-\frac{(1-\delta)}{\gamma}\right) i_t^* \tag{11}$$

$$+ \gamma^2\,\varphi\left(1-\frac{(1-\delta)}{\gamma}\right)\left(1+\beta\,\gamma^{(1-\sigma_c)}\right)\varepsilon_t^i$$

Sticky Wage economy:

$$
\begin{aligned}
\mu_t^p &= \alpha\, r_t^k + (1-\alpha)\; w_t - \varepsilon_t^a && (12)\\
z_t &= \frac{1}{\frac{\psi}{1-\psi}}\, r_t^k && (13)\\
r_t^k &= w_t + l_t - k_t^s && (14)\\
k_t^s &= z_t + k_{t-1} && (15)\\
i_t &= \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\; \left(i_{t-1} + \beta\gamma^{(1-\sigma_c)} E_t\, i_{t+1} + \frac{1}{\gamma^2\,\varphi}\, q_t \right) + \varepsilon_t^i && (16)\\
q_t &= -r_t + E_t\pi_{t+1} + \left(1 - \beta\,(1-\delta)\,\gamma^{-\sigma_c}\right)\; E_t r_{t+1}^k && (17)\\
&\quad + \beta\,(1-\delta)\,\gamma^{-\sigma_c}\; E_t q_{t+1} - \varepsilon_t^b\\
c_t &= \frac{\frac{h}{\gamma}}{1+\frac{h}{\gamma}}\, c_{t-1} + \frac{1}{1+\frac{h}{\gamma}} E_t\, c_{t+1} + \frac{(\sigma_c - 1)\; w^{ss} l^{ss}}{c^{ss}\sigma_c\, \left(1+\frac{h}{\gamma}\right)}\; (l_t - E_t l_{t+1}) && (18)\\
&\quad - \frac{1-\frac{h}{\gamma}}{\sigma_c\, \left(1+\frac{h}{\gamma}\right)}\; (r_t - E_t\pi_{t+1}) + \varepsilon_t^b\\
y_t &= c_y\, c_t + i_i\, i_t + r^{kss} k_y\, z_t + \varepsilon_t^g && (19)\\
y_t &= \Phi\; (\alpha\, k_t^s + (1-\alpha)\; l_t + \varepsilon_t^a) && (20)\\
r_t &= \; r_\pi\; (1-\rho)\,\pi_t + (1-\rho)\; r_y\; (y_t - y_t^*) && (21)\\
&\quad + r_{\Delta y}\; \left(y_t - y_t^* - y_{t-1} + y_{t-1}^*\right) + \rho\, r_{t-1} + \varepsilon_t^r\\
\pi_t &= \frac{1}{1+\beta\gamma^{(1-\sigma_c)}\,\iota_p} * && (22)\\
&\quad \left(\beta\gamma^{(1-\sigma_c)} E_t\pi_{t+1} + \iota_p\, \pi_{t-1} + \frac{\frac{(1-\xi_p)\,\left(1-\,\beta\gamma^{(1-\sigma_c)}\xi_p\right)}{\xi_p}}{1+(\Phi-1)\;\varepsilon_p}\, \mu_t^p \right) + \varepsilon_t^p\\
w_t &= \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}\; w_{t-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}}\; E_t w_{t+1} + \frac{\iota_w}{1\beta\gamma^{(1-\sigma_c)}}\pi_{t-1} && (23)\\
&\quad - \frac{1+\beta\gamma^{(1-\sigma_c)}\,\iota_w}{1+\beta\gamma^{(1-\sigma_c)}}\pi_t \;+\; \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}} E_t\pi_{t+1}\\
&\quad + \frac{(1-\xi_w)\;\left(1-\beta\gamma^{(1-\sigma_c)}\,\xi_w\right)}{\left(1+\beta\gamma^{(1-\sigma_c)}\right)\;\xi_w}\; \frac{1}{1+(\lambda_w - 1)\;\varepsilon_w} *\\
&\quad \left(\sigma_l\, l_t + \frac{1}{1-\frac{h}{\gamma}}\, c_t - \frac{\frac{h}{\gamma}}{1-\frac{h}{\gamma}}\, c_{t-1} - w_t \right) + \varepsilon_t^w\\
k_t &= \frac{(1-\delta)}{\gamma}\, k_{t-1} + \left(1 - \frac{(1-\delta)}{\gamma}\right)\; i_t && (24)\\
&\quad + \varphi\,\gamma^2\; \left(1 - \frac{(1-\delta)}{\gamma}\right) \left(1 + \beta\,\gamma^{(1-\sigma_c)}\right) \varepsilon_t^i
\end{aligned}
$$

Dynamics of the shocks:

$$\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a \tag{25}$$

$$\varepsilon_t^b = \rho_b \, \varepsilon_{t-1}^b + \eta_t^b \tag{26}$$

$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga} \eta_t^a \tag{27}$$

$$\varepsilon_t^i = \rho_I \, \varepsilon_{t-1}^i + \eta_t^I \tag{28}$$

$$\varepsilon_t^r = \rho_r \, \varepsilon_{t-1}^r + e_t^r \tag{29}$$

$$\varepsilon_t^p = \rho_\pi \, \varepsilon_{t-1}^p + \eta_t^p - \mu_p \, \eta_{t-1}^p \tag{30}$$

$$\varepsilon_t^w = \rho_w \, \varepsilon_{t-1}^w + \eta_t^w - \mu_w \, \eta_{t-1}^w \tag{31}$$

Measurement equations:

$$dlGDP_t = y_t - y_{t-1} + \bar{\gamma} \tag{32}$$

$$dlCONS_t = \bar{\gamma} + c_t - c_{t-1} \tag{33}$$

$$dlINV_t = \bar{\gamma} + i_t - i_{t-1} \tag{34}$$

$$dlWAG_t = \bar{\gamma} + w_t - w_{t-1} \tag{35}$$

$$dlP_t = \pi_t + \bar{\pi} \tag{36}$$

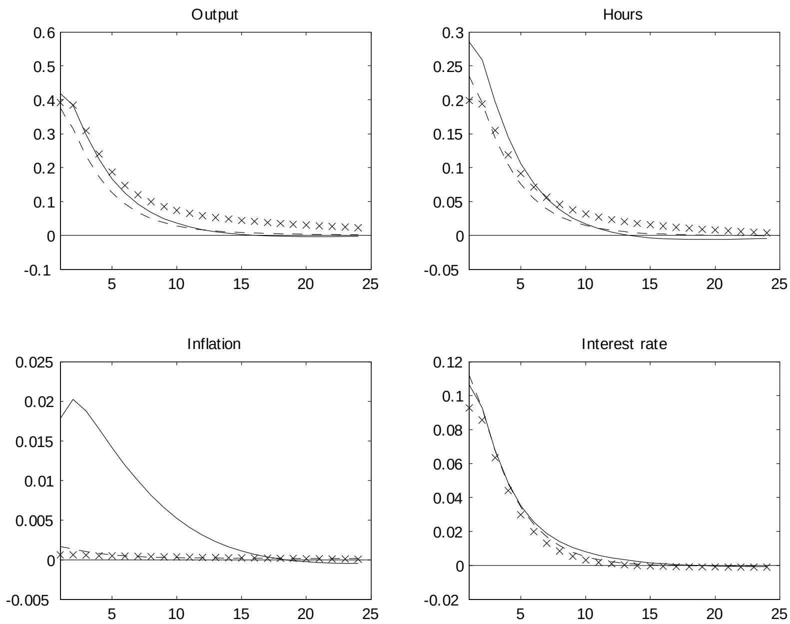
$$FEDFUNDS_t = r_t + \bar{r} \tag{37}$$

$$lHOURS_t = l_t + \bar{l} \tag{38}$$

2 Smets and Wouters (2007): comparison between IRFs obtained with Bayesian and Maximum Likelihood estimation

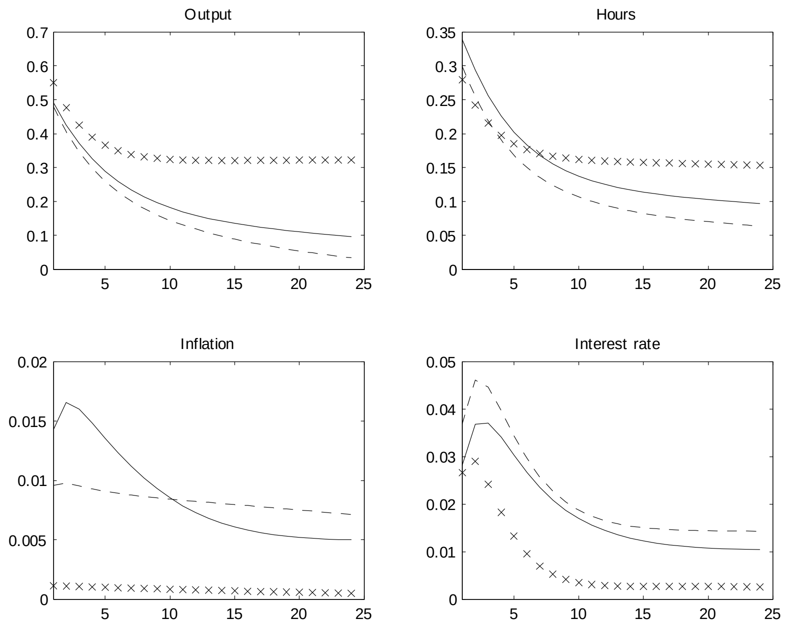
In the following figures we report the impulses responses by Smets and Wouters (in solid line), the ones we obtained by estimating the model with ML on the same sample (in dashed line), and on the sample extended to the end of 2007 (in x-marked line). The graphs are meant to show that the results obtained with ML are fairly similar to the original ones with Bayesian techniques. Moreover, our ML results on the original sample are very similar to the ones obtained by Iskrev (2008) in a similar experiment.

Figure 1: Impulse Responses to a risk-premium shock



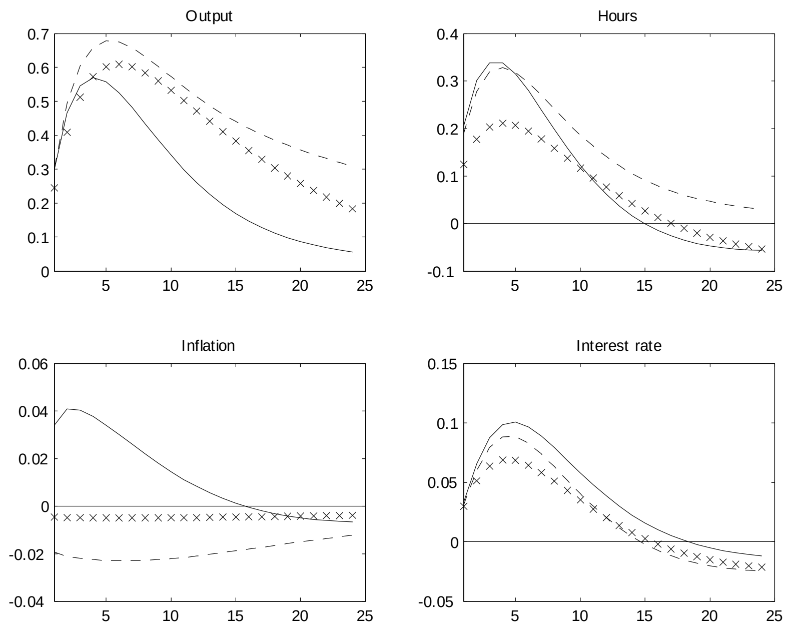
Notes: The impulse responses in solid lines are those obtained by Smets and Wouters (2007). The dashed ones, are those obtained with ML estimation on the same sample. The x-marked ones are those obtained with ML on the sample extended to 2007.

Figure 2: Impulse Responses to an exogenous spending shock



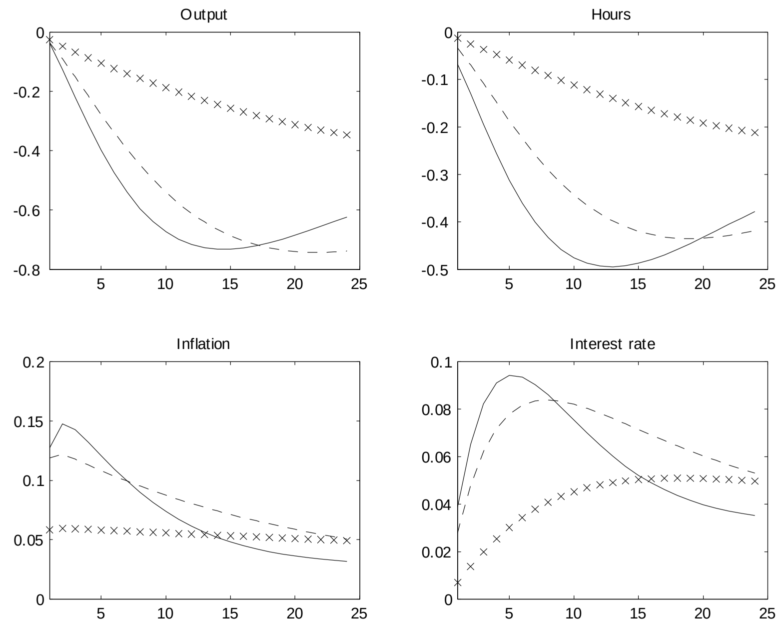
Notes: See Notes at Figure 1.

Figure 3: Impulse Responses to an investment-specific technology shock



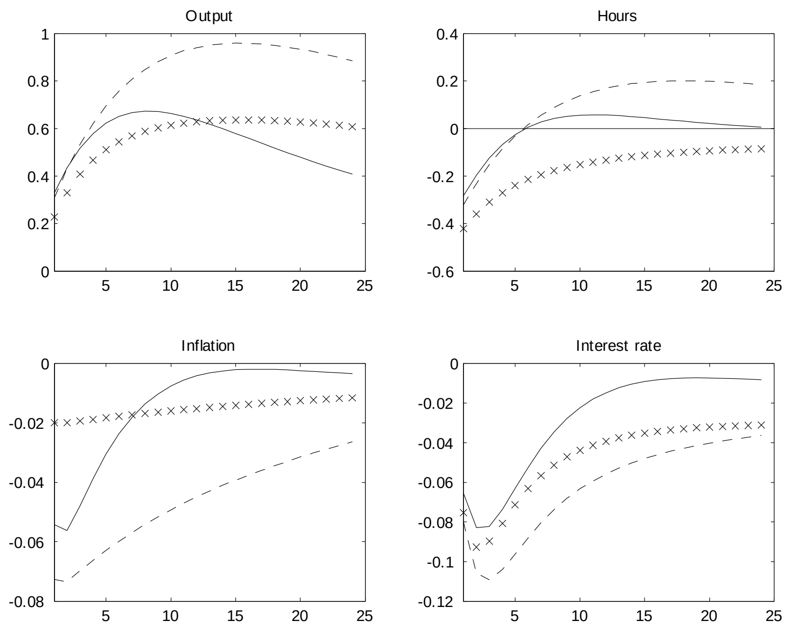
Notes: See Notes at Figure 1.

Figure 4: Impulse Responses to a wage markup shock



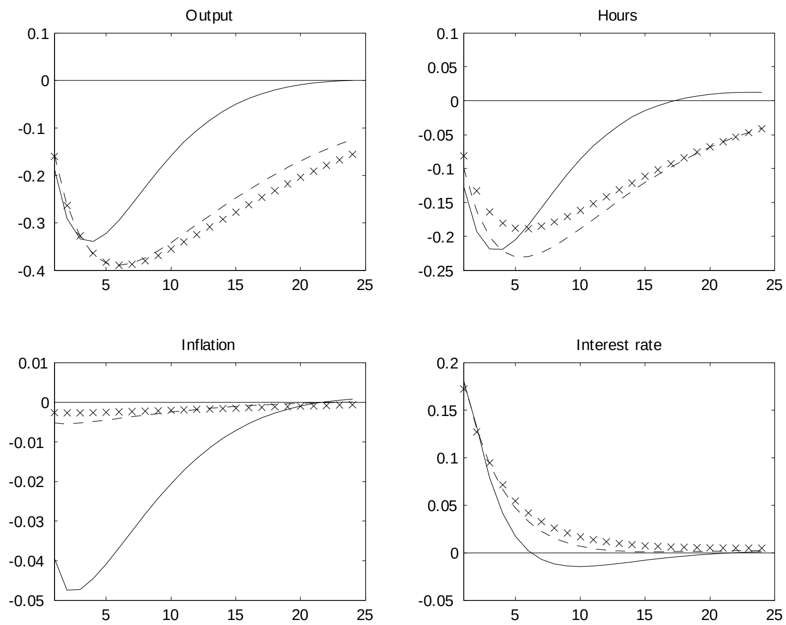
Notes: See Notes at Figure 1.

Figure 5: Impulse Responses to a productivity shock



Notes: See Notes at Figure 1.

Figure 6: Impulse Responses to a monetary policy shock



Notes: See Notes at Figure 1.