Online Appendix to Declining Discount Rates in Singapore's Market for Privately Developed Apartments

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A.1 Sample coverage: New condominium apartments

The REALIS database comprises all private residential property transactions for which caveats were lodged with the Singapore Land Authority. A caveat is a legal document lodged by a buyer or his mortgage provider to protect his interests in that property by preventing contradictory dealings with the property from being registered. Only a minor proportion of transactions do not see a caveat lodged. In particular, buyers who do not take out a mortgage are not required to lodge a caveat, although many still choose to.

In addition to missing some transactions without lodged caveats, we exclude 214 multi-unit transactions, which account for 1% of our sample. We do so because we do not have information on these apartments' individual characteristics, such as apartment size and story. For projects with missing completion year in REALIS, we consulted the real-estate websites propertyguru.com.sg, iproperty.com.sg, and stproperty.sg. We failed to find the completion year for 19 apartments and exclude their sales from our analysis. We also exclude 28 transactions that have missing story (or are basement units), and another 26 units sold 11 years after construction was completed; these likely relate to used units.

We validate our sample coverage by comparing the number of units that appear in our sales data with the total number of constructed units. The latter information is available from REALIS for condominium projects with construction completed in the second quarter of 2002 or later. Since it might take the developer several years to sell all units in a project, here we focus on the 735 projects that were completed between 2003 and 2012 and have at least one recorded transaction. Out of a universe of 82,991 constructed units for these 735 projects, we observe the sale of 77,095 units in our transaction data, or 93% of the population. Another way to check our sample coverage is to compare the changes in the stock of condominium units reported by the Urban Redevelopment Authority against the number of new condominium transactions in our sample, as we do in the text.

A.2 Location fixed effects

Singapore uses a 6-digit postal code system in which a 6-digit code identifies a building. Thus, a lease-invariant condominium project with five apartment buildings will have five 6-digit codes, one for each building. We face a trade-off when specifying the granularity of our geographic controls.

Specifying location fixed effects that are too detailed absorbs the variation in remaining lease length that we seek to exploit. For an extreme example, consider the 6-digit level, with zero variation in lease contract. Consider a condominium project on a lease of "99 Yrs From 2/11/2009" and realistically suppose that all units in one of its 6-digit buildings were sold before construction was completed (and housing benefits began) in 2013: All units would have the same remaining lease of 95 years. Moreover, too tight locations result in a less favorable number of transactions per fixed effect, with potentially larger incidental parameter bias. On the other hand, specifying geographic controls that are too coarse can lead to inconsistent estimates of the effect of tenure on value, because tenure might correlate with unobserved quality differences associated with location.

To allow for enough variation in lease type while controlling for local neighborhood characteristics, consider grouping properties by the first three digits of the postal code. Doing so results in 187 3-digit postal areas, 108 of which have properties of more than one lease type—i.e., perpetual leases, multi-century leases, and multi-decade leases. In the 79 areas in which there is no variation in lease type, e.g., all apartments are multi-decade leases, 30 areas contain only one condominium project.

Given that the proportion of Singapore's land occupied by residences was 14%, totaling 100 km^2, dividing this residential land into 187 regions enables us to control for geographic variation at quite a granular level.[1] To demonstrate this, we obtained the geographic coordinates for each of the 1,672 condominium projects in our sample and calculated the bilateral distance between each pair of projects that share the same 3-digit postal area. The distribution of these within-area bilateral distances is plotted in Figure A.2. Across pairs of properties with the same 3-digit location, the majority are located in close proximity, for example the neighboring Botannia and Infiniti located in area 127xxx. Specifically, 75% of these bilateral distances are shorter than 1 km and 95% of the distances are shorter than 1.8 km. In other words, 95% of the projects with a common 3-digit control are located in a small area with a 0.9 km radius. Only 8 projects (44 bilateral distances) are located more than 4 km from another project with a 3-digit area in common.

Specifying 3-digit location fixed effects may provide a good compromise between controlling for potentially confounding spatial heterogeneity and preserving variation in tenure, which is the object of interest. At the same time, we implement specifications with more granular 5-digit controls.

[1] Land use in 2010 (www.mnd.gov.sg/landuseplan/). Notice that $100/187 = 0.53$ km^2 per 3-digit fixed effect, i.e., less than a "unit square" of side 1 km. Housing in Singapore agglomerates in specific local neighborhoods around different axes from the original downtown area.

A.3 Adjusting prices, including prepayments, to 2014 S$

Payment of the purchase price is due in full when a buyer purchases a condominium apartment that is already completed. In this case, payment coincides with the start of housing benefits (and we adjust the historical purchase price to January 2014 S$). When a buyer purchases property that is still under development, partial payment is made prior to the start of housing benefits. Here, we take the observed time difference between purchase and construction completion, and compound (upward adjust) the transaction price using a typical payment scheme that developers follow for sales during construction. In particular, we adopt the scheme recommended under current Urban Redevelopment Authority guidelines. There has been little variation in prepayment schemes over the sample period and across developers.

Under the Urban Redevelopment Authority scheme, 20% of the purchase price is due upon signing the Sale & Purchase agreement, or within 8 weeks from the Option to Purchase. Another 40% is due in gradual payments based on completion of six staggered stages of construction. For example, 10% is due when the foundation is completed. Another 10% is due when the concrete framework is completed. At a subsequent stage, 25% is due when the Temporary Occupation Permit or Certificate of Statutory Completion is issued, and the final 15% is due soon after, upon construction completion. These last two payments are typically due within 1 year of each other.

We approximate this payment scheme and adjust the transaction price as follows. For a purchase observed any year before the condominium's completion year, we adjust any amount due at the time of purchase for milestones that had already passed—e.g., the initial 20% deposit and any payments such as the 10% for foundation work—using the CPI of the purchase month-year, and adjust to January 2014 S$. The 40% of the price due at or near completion is adjusted by the completion-year June CPI. (We do not observe the month of completion and take it to be June, as a midpoint.) Other intermediate payments—e.g., the 10% for concrete work—are adjusted by the (midpoint) June CPI of the years between the purchase year and the completion year.

How the remaining 60% beyond the 40% due in the completion year is adjusted depends on the number of years that elapse from purchase to construction completion. For an apartment purchased in March, for instance, 1 year before the completion year, we adjust the remaining 60% by the purchase-year March CPI. For a purchase in March 2 years before completion, we adjust 50% of the price by the purchase-year March CPI and 10% by the year-after-purchase June CPI.

For a purchase in March 3 years before completion, we adjust 40% of the price by the purchase-year March CPI, 10% by the year-after-purchase June CPI, and 10% by the 2-years-after-purchase June CPI. Finally, for a purchase 4 years or more before June of the completion year, we adjust 20% of the price by the purchase month-year CPI, 20% by the year-after-purchase June CPI, 10% by the 2-years-after-purchase June CPI, and 10% by the 3-years-after-purchase June CPI.

Again, any payments due at the time of purchase are adjusted by the CPI of the purchase month and year to January 2014 S$. For example, for a purchase in March, say, either in the year of or after construction completion, we adjust the entire purchase price by the March CPI of the purchase year, since payment would have been due in full at the time of the purchase.

A.4 Model implementation

Functional forms for the discount rate schedule

In addition to modeling the discount rate schedule as an exponential function of time, we fit a logarithmic function and a hyperbolic function of time, respectively:

$$r_t = r(t;\gamma) = \begin{cases} \max\left(\gamma_1 + \gamma_2 \log t, \gamma_3\right) & \text{for } 1 < t \leq 10^6 \\ \max\left(\gamma_1 + \gamma_2 \log 10^6, \gamma_3\right) & \text{for } t > 10^6 \end{cases} \qquad \text{(logarithmic)}$$

$$r_t = r(t;\gamma) = \begin{cases} \max\left(\gamma_1 t^{\gamma_2}, \gamma_3\right) & \text{for } 1 < t \leq 10^6 \\ \max\left(\gamma_1 \left(10^6\right)^{\gamma_2}, \gamma_3\right) & \text{for } t > 10^6 \end{cases} \qquad \text{(hyperbolic)}$$

As stated in the text, across all parametric structures, slope parameter γ_2 is the key one of interest, γ_1 is the period-1 discount rate, and γ_3 bounds the discount rate from below. A negative (resp., positive) γ_2 implies declining (resp., rising) discount rates, and a zero value for this parameter implies a constant discount rate.

Trend acceleration penalty

We add a rate acceleration penalty to the objective function when estimating r_t with multiple steps (Tables 4 and A.11). (In principle, we could similarly introduce even more smoothness to the parametric structures as well.) With steps one century wide until the tenth century, the sum of squared second differences (SSSD) in the penalized NLLS criterion function (6) is

$$\sum_{t=1}^{8}\left((r_{t+2}-r_{t+1})-(r_{t+1}-r_t)\right)^2,$$

where $r_1, r_2, , ..., r_9$ denote annual discount rates over the first, second, ..., ninth century and r_{10} denotes the annual discount rate over the tenth and subsequent centuries (year 901 on). With steps one century wide until the fifth century, we further restrict the discount rate for the sixth and subsequent centuries to equal that in the fifth century, i.e., $r_5 = r_6 = ... = r_{10}$. In practice, we choose a smoothing parameter λ such that the rate acceleration penalty $\lambda\times$SSSD accounts for only a small fraction of the optimized objective function, e.g., $\lambda\times$SSSD=0.04 out of RSS+$\lambda\times$SSSD= 1513.43, in Table 4, column 3.

Model cross validation

Cross validation is a common model evaluation and selection tool in the machine learning and statistical learning toolbox (James et al. (2013), pp.176-186). Essentially, the approach splits a dataset into an estimation sample (the training set) to estimate the model and then predicts the dependent variable in a held-out sample (the test set) using the estimated model. The out-of-sample residuals are then used to compute the Mean Squared Error (MSE). Often, the model with the lowest MSE is judged the better predictive model.

We also make use of cross validation, treating it as another tool to compare the relative performance of the various models we estimate, without explicitly using it to pick a definitive model (thought the reader could do so if desired). Unlike the traditional cross-validation approach, we also conduct pairwise t-tests of the difference in the MSEs of the various models.

Generally, the cross-validation MSE is computed as follows:

1. Randomly sort the N observations.

2. Split the observations into K groups of roughly equal size.

3. For $k = 1, \ldots, K$,

   (a) Exclude the N_k observations in group k. The N_{-k} observations in the remaining $K-1$ groups make up the training set.

   (b) Estimate the model on the training set.

(c) Use the estimated model to calculate the (out-of-sample) residuals $\tilde{\varepsilon}_i$ for the observations in group k, which make up the test set.

4. Compute, across N observations in all K groups, $\text{MSE} = \frac{1}{N}\sum_{i=1}^{N} \tilde{\varepsilon}_i^2$.

We adjust the procedure described above because of our use of location fixed effects. To ensure that the same locations are contained in the training and test sets at each iteration k of step 3, we amend steps 1 and 2 by randomly partitioning the observations within location. We omit observations for any location that has fewer than 10 observations, i.e., less than one observation per fold given a choice of $K = 10$ folds. Cross validation of the models with 5-digit location controls includes 178,588 observations and with 3-digit location controls, 179,202 observations (down slightly from the 179,218 observations in the full sample).

Common choices for the number of folds K are 5, 10, or 20 (Hansen (2020), p.860). Our results are robust to the choice; we report the results of using 10 folds. (Figure A.8 illustrates the stability of the discount rate schedule parameter estimates across training sets.)

We perform pairwise equality tests within and between the models reported in Tables 2 and 4. Under the null hypothesis that the MSEs of models A and B are equal, the t-statistic is:

$$t = \frac{\frac{1}{N}\sum_{i=1}^{N}\left(\tilde{\varepsilon}_{A,i}^2 - \tilde{\varepsilon}_{B,i}^2\right)}{\sqrt{\hat{\sigma}^2/N}}, \tag{1}$$

where $\tilde{\varepsilon}_{A,i}$ is observation i's out-of-sample residual from model A, $\tilde{\varepsilon}_{B,i}$ is observation i's out-of-sample residual from model B, $\hat{\sigma}^2$ is the sample variance of $\tilde{\varepsilon}_A^2 - \tilde{\varepsilon}_B^2$, and N is the number of observations in the cross-validation exercise.

To compute the MSE for model (2) in which the dependent variable is log price, we transform the log price prediction to a prediction of price, and take the difference relative to observed price as the residual $\tilde{\varepsilon}_i$. After squaring and taking the mean across observations, this MSE for the log price model can then be compared to MSE for model (3) in which the dependent variable is price.

Specifically,

$$
\begin{aligned}
\mathrm{E}\left(p_{im} \mid X_i, L_i, \theta, \mathbf{r}, \xi_m\right) &= \mathrm{E}\left(e^{\ln p_{im}} \mid X_i, L_i, \theta, \mathbf{r}, \xi_m\right), \\
&= \mathrm{E}\left(e^{\ln u(X_i;\theta)+\ln \phi(L_i;\mathbf{r})+\xi_m+\varepsilon_{im}} \mid X_i, L_i, \theta, \mathbf{r}, \xi_m\right), \\
&= e^{\ln u(X_i;\theta)+\ln \phi(L_i;\mathbf{r})+\xi_m}\ \mathrm{E}\left(e^{\varepsilon_{im}} \mid X_i, L_i, \theta, \mathbf{r}, \xi_m\right), \\
&= e^{\ln u(X_i;\theta)+\ln \phi(L_i;\mathbf{r})+\xi_m}\ \mathrm{E}\left(e^{\varepsilon_{im}}\right).
\end{aligned}
$$

noting, in the last step, the idiosyncratic mean-zero shock ε_{im}. We estimate $\mathrm{E}\left(e^{\varepsilon_{im}}\right)$ as

$$
\widehat{\mathrm{E}\left(e^{\varepsilon_{im}}\right)} = \frac{1}{N_k}\sum_{i=1}^{N_k} e^{\tilde{\varepsilon}_{im}} = \frac{1}{N_k}\sum_{i=1}^{N_k} e^{\ln p_{im}-\ln u\left(X_i;\hat{\theta}\right)-\ln \phi(L_i;\hat{\mathbf{r}})-\hat{\xi}_m}
$$

(Here the hat denotes parameter estimates implemented on a training set of size N_{-k} and predictions in a test set of size N_k.) A prediction of price p_{im} from log price model (2) is then:

$$
\hat{p}_{im} = e^{\ln u\left(X_i;\hat{\theta}\right)+\ln \phi(L_i;\hat{\mathbf{r}})+\hat{\xi}_m}\ \widehat{\mathrm{E}\left(e^{\varepsilon_{im}}\right)}
$$

A.5 Rental prices and unobserved heterogeneity

Our purchase price regressions control for many observed property characteristics such as location, apartment size, and story, in addition to age of construction directly by choice of sample. Here, we look to the rental market for an indication of whether unobserved quality differs systematically by (landlord) tenure length. Since renters should care only about flows of housing services and not differences in the landlord's ownership tenure, finding that tenure has no effect on rental prices provides evidence that apartments of different lease types do not differ in unobserved quality.

The Urban Redevelopment Authority reports rental contracts for privately developed apartments submitted to the Inland Revenue Authority of Singapore for Stamp Duty assessment (see next section) within the last 36 months at the time of the search (February 2012 to January 2015). The data include monthly rent and starting month of the rental contract, the condominium project name and street, the apartment's floor area in mutually exclusive categories, and the number of bedrooms. The apartment building—equivalently, 6-digit postal code—is not reported, and buildings in the same project may span different 5-digit codes. We merged project-level information, such as lease type, construction completion year, total number of apartments, and maximal story, from the REALIS transaction sales data.

To control for unobserved heterogeneity arising from differences in renovation, maintenance, and depreciation, and in line with our analysis of property purchase prices, we restrict the sample to rentals of condominium apartments that were aged less than 10 years at the time of rental. Given some extreme prices in the data, we trimmed the top and bottom 1% of the rental price distribution. The data do not distinguish between studio units and a multi-bedroom unit renting out one bedroom, so we drop single-bedroom contracts. The final sample consists of 42,699 rental contracts, of which 58%, 5%, and 37% are apartments for which landlords have perpetual, multi-century, and multi-decade leases, respectively.

Table A.12 reports estimates of OLS regressions with the log of the apartment's monthly rental price (in January 2014 S$) as the dependent variable. Estimates on the multi-century and multi-decade landlord leases are small and statistically insignificant (the dummy for perpetual landlord leases is the reference category). With very tight 5-digit location controls, any rental price difference between perpetual and multi-century landlord leases cannot be identified. Reflecting the tightness of 5-digit controls, there are only four 5-digit locations where rental contracts on multi-century landlord leases and at least one more landlord lease type are observed; this number of "multi-century plus" locations climbs from 4 to 28, i.e., sevenfold, with less tight but still quite fine 3-digit controls. In sum, evidence from the rental market suggests that unobserved quality across apartments of varying lease types is unlikely to be strong.

A.6 Restrictions on publicly developed apartments and taxes

Apartments developed through the Housing Development Board (HDB)

Compared to the largely unrestricted market for privately developed housing, the government regulates the purchase of new and used HDB apartments. We describe some key HDB regulations.

Only Singapore citizens can buy new HDB apartments, and HDB resales can be purchased by citizens and permanent residents. The price of new HDB apartments is heavily discounted. As many as 13 different schemes provide additional subsidies. For example, the "Additional Central Provident Fund Housing Grant" and the "Special Central Provident Fund Housing Grant" each provide up to S$ 40,000 in additional subsidies to eligible buyers. Along with these subsidies is an income ceiling, as the government attempts to push buyers to privately developed housing. For example, average gross monthly household income cannot exceed S$ 12,000 to be eligible to buy an HDB apartment with 4 rooms or more. These schemes come with other restrictions as well. For example, the most common scheme, the Public Scheme, targets families. The buyer must be at least 21 years old and either be married, living with parents, or, in the case of widowed or divorced persons, have legal custody of children.

New HDB apartments have a minimum occupancy period of 5 years. There is no occupancy requirement for privately developed housing, but property is subject to extra taxes if resold within 4 years of the date of purchase, presumably to curb "flipping." For example, selling a unit within 1 year of purchase is subject to an additional 16% tax.

Stamp duty

Two types of ad valorem duties are payable by buyers: the Buyer's Stamp Duty (BSD), and the Additional Buyer's Stamp Duty (ABSD). The BSD rate is 1% for the first S$ 180,000, 2% for the next S$ 180,000, and 3% for the remaining amount. The ABSD was introduced in December 2011 to curb property price growth, and raised in January 2013. The schedule of the ABSD is as follows.

Buyer's Residency Status	Dec/2011 to Jan/2013	After Jan/2013
Citizens, 1st property	Not applicable	Not applicable
Citizens, 2nd property	Not applicable	7%
Citizens, 3rd and subsequent property	3%	10%
Permanent Residents, 1st property	Not applicable	5%
Permanent Residents, 2nd and after	3%	10%
Foreigners, any property	10%	15%

Since there are only 314 observations with a transaction price below S$ 360,000, the combined BSD/ABSD marginal tax rate depends only on the buyer's residency status, not on the value of the property. After the introduction of ABSD in 2011, it is possible that the increased progressivity of stamp duty on some non-citizen buyers—and foreigners in particular—may have dampened demand for higher-value properties. To the extent that this shift adversely affected the demand for perpetual leases over comparable lower-value maturities, controlling for this shift would increase the estimated perpetuity premium even further.

Property tax

Annual property tax is an increasing function of a property's Annual Value (AV). The Inland Revenue Authority of Singapore determines AV from market rental prices for rented-out units in the same or comparable condominium projects (rent depends on the flow utility of housing services, but not on landlord's lease). For most of our sample period, until January 2014, the property tax rate for owner-occupied units was 0 for the first S$ 6,000, 4% for the next S$ 59,000, and 6% for the amount exceeding S$ 65,000. Over the last few months in our sample, after January 2014—which we drop in a robustness test—property taxes for owner-occupied units became more progressive: 0 for the first S$ 8,000, 4% for the next S$ 47,000, and the rate increases by 2% for every S$ 15,000 increase in Annual Value until it reaches the maximum of 16%. For non-owner-occupied units, the marginal tax rate is a flat 10%.

References

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Table A.1: Selected descriptive statistics by lease type.

				Percentile						
Variable	N	Mean	Std. Dev.	1st	5th	25th	50th	75th	95th	99th
Transaction price (S$)										
Perpetual leases	72,834	1,607,528	1,521,039	493,670	583,191	885,001	1,184,122	1,711,683	4,051,760	7,681,821
Multi-century leases	9,154	1,468,902	1,308,227	540,917	678,796	958,601	1,220,180	1,532,410	2,581,755	7,212,714
Multi-decade leases	97,230	1,120,668	757,722	483,220	594,389	781,703	959,496	1,214,427	2,099,475	3,687,873
Total	179,218	1,336,314	1,181,158	490,660	591,724	819,090	1,042,092	1,408,670	2,976,940	6,403,549
Price per m^2 (S$)										
Perpetual leases	72,834	14,122	6,315	6,197	7,500	9,752	12,517	16,760	25,635	36,692
Multi-century leases	9,154	12,349	4,860	6,475	7,456	9,215	10,909	14,588	21,338	34,362
Multi-decade leases	97,230	11,067	4,504	5,465	6,299	8,128	9,969	12,863	19,535	27,467
Total	179,218	12,374	5,530	5,622	6,589	8,786	10,848	14,518	22,988	32,308
Time from sales to delivery (years)										
Perpetual	72,834	-2.6	1.5	-5	-5	-4	-3	-2	0	1
Multi-century	9,154	-1.9	1.4	-4	-4	-3	-2	-1	1	2
Multi-decade	97,230	-2.1	1.5	-5	-4	-3	-2	-1	0	1
Total	179,218	-2.4	1.5	-5	-5	-3	-2	-1	0	1
Buyer age (years)										
Perpetual leases	25,423	42.4	9.8	25.5	29.0	35.0	41.0	48.5	60.0	69.5
Multi-century leases	3,120	41.9	9.3	25.0	29.0	35.0	40.5	48.0	59.5	66.0
Multi-decade leases	26,005	41.9	9.3	25.0	29.0	35.0	41.0	48.0	59.0	67.0
Total	54,548	42.1	9.5	25.5	29.0	35.0	41.0	48.0	59.5	68.0

Notes: Purchases of new privately developed apartments from 1995 to 2015 ($N = 179,218$). We observe buyer age for a subset of transactions. See the notes to Table 1. Figure A.3 further describes characteristics of transacted apartments by lease type.

Table A.2: OLS regressions with tenure grouped into bins.

Sample:	Full sample: All areas			Areas w/ longer lease types		
Location controls:	5-digit	4-digit	3-digit	5-digit	4-digit	3-digit
Dependent variable: Log price	(1a)	(2a)	(3a)	(4a)	(5a)	(6a)
825 to 986 years (1=yes)	0.007	0.003	-0.035**	-0.029**	-0.002	-0.048***
	(0.017)	(0.020)	(0.015)	(0.014)	(0.016)	(0.015)
87 to 99 years (1=yes)	-0.159***	-0.160***	-0.148***	-0.290***	-0.252***	-0.183***
	(0.019)	(0.019)	(0.012)	(0.037)	(0.037)	(0.021)
56 to 63 years (1=yes)	-0.427***	-0.360***	-0.369***			
	(0.017)	(0.021)	(0.029)			
R^2	0.942	0.903	0.857	0.943	0.903	0.859
Dependent variable mean	9.345	9.345	9.345	9.412	9.412	9.412
Dependent var.: Price per m^2	(1b)	(2b)	(3b)	(4b)	(5b)	(6b)
825 to 986 years (1=yes)	0.095	-0.062	-0.645**	-0.368	-0.104	-0.849***
	(0.259)	(0.365)	(0.261)	(0.285)	(0.295)	(0.273)
87 to 99 years (1=yes)	-1.858***	-1.742***	-1.622***	-3.437***	-2.591**	-1.867***
	(0.353)	(0.447)	(0.276)	(0.969)	(0.826)	(0.479)
56 to 63 years (1=yes)	-6.269***	-5.118***	-5.163***			
	(0.362)	(0.338)	(0.416)			
R^2	0.926	0.855	0.775	0.927	0.842	0.775
Dependent variable mean	12.374	12.374	12.374	13.262	13.262	13.262
5-digit location fixed effects	Yes (1516)			Yes (290)		
4-digit location fixed effects		Yes (630)			Yes (111)	
3-digit location fixed effects			Yes (187)			Yes (25)
Locations w/ single lease type	96%	82%	42%	0%	0%	0%
Number of regressors	1558	672	229	331	152	66

Notes: This table shows estimates for 12 OLS regressions. In the top panel (columns 1a-6a), the dependent variable is the log of the apartment's transaction price per square meter of floor area. In the bottom panel (columns 1b-6b), the dependent variable is the apartment's transaction price per square meter of floor area (1000 S$/$m^2$). Specifications 1-3 use the full sample of 179,218 new apartment purchases from 1995 to 2015, across all areas. Specifications 4-6 use the subsample of 31,072 transactions in areas in which at least multi-century and perpetual leases were traded in sample. In all specifications, the dummy for perpetual leases is the reference category. We vary the granularity of location controls, from 5 digit to 3 digit, as indicated. With 5-digit controls, 96% of 1516 locations (e.g., 12772x) contain a single lease type (multi-decade, multi-century or perpetual). With 3-digit controls, a lower 42% of 187 locations (e.g., 127xxx) contain a single lease type. Besides location, all specifications control for apartment size and its square; apartment story and its square; an indicator for apartment on 1st story; an indicator for apartment on top story (and interactions with apartment size and its square); project size and its square; a project-specific indicator for a large land parcel; purchase-to-completion bins of width 1 year; year-of-purchase fixed effects; and quarter-of-year fixed effects. Standard errors, in parentheses, are clustered by building. Two-way clustering by building and purchase year yields similar values. *** p<0.01, ** p<0.05, * p<0.1.

Table A.3: OLS regressions with tenure grouped into bins: Additional controls.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
With 5-digit location controls:							
825 to 986 years (1=yes)	0.008	0.007	0.010	0.001	0.007	0.007	-0.016
	(0.017)	(0.017)	(0.018)	(0.017)	(0.017)	(0.018)	(0.019)
87 to 99 years (1=yes)	-0.154***	-0.160***	-0.180***	-0.174***	-0.160***	-0.156***	-0.165***
	(0.019)	(0.019)	(0.023)	(0.018)	(0.019)	(0.020)	(0.016)
56 to 63 years (1=yes)	-0.432***	-0.425***	-0.402***	-0.453***	-0.423***	-0.420***	-0.623***
	(0.018)	(0.017)	(0.018)	(0.016)	(0.017)	(0.022)	(0.030)
R^2	0.944	0.942	0.945	0.949	0.942	0.942	0.945
Number of regressors	1634	1624	1682	1615	1566	1525	1176
With 3-digit location controls:							
825 to 986 years (1=yes)	-0.034**	-0.035**	-0.057***	-0.042***	-0.034**	-0.035**	-0.025
	(0.015)	(0.015)	(0.013)	(0.014)	(0.015)	(0.015)	(0.017)
87 to 99 years (1=yes)	-0.142***	-0.148***	-0.162***	-0.144***	-0.148***	-0.148***	-0.150***
	(0.012)	(0.012)	(0.010)	(0.011)	(0.012)	(0.012)	(0.014)
56 to 63 years (1=yes)	-0.399***	-0.367***	-0.294***	-0.379***	-0.372***	-0.374***	-0.436***
	(0.029)	(0.029)	(0.047)	(0.028)	(0.029)	(0.028)	(0.077)
R^2	0.862	0.858	0.875	0.873	0.857	0.857	0.866
Number of regressors	305	295	337	286	237	229	211
Apartment size bins	Yes						
Apartment story bins		Yes					
Location control × large land parcel			Yes				
Year-quarter fixed effects				Yes			
Month-of-year fixed effects					Yes		
Shared property amenities						Yes	
Distance to shopping mall (in log)						Yes	
Buyer's age (years)							Yes

Notes: This table shows estimates for 14 OLS regressions. The dependent variable is the log of the apartment's transaction price per square meter of floor area. All specifications use the full sample of 179,218 new apartment purchases (or 177,689 and 54,548 observations in specifications 6 and 7 due to missing values for amenities and buyer age, respectively). In all specifications, the dummy for perpetual leases is the reference category. Across panels, we vary the granularity of location controls, from 5 digit to 3 digit. Besides location, and unless noted otherwise, all specifications control for: apartment size and its square; apartment story and its square; an indicator for apartment on 1st story; an indicator for apartment on top story (and interactions with apartment size and its square); project size and its square; a project-specific indicator for a large land parcel; purchase-to-completion bins of width 1 year; year-of-purchase fixed effects; and quarter-of-year fixed effects. Specification 1 replaces apartment size and its square by apartment size bins of width 10 m^2. Specification 2 replaces apartment story and its square by a series of individual story dummies. Specification 3 adds interactions between the large land parcel indicator and the location fixed effects. Specification 4 replaces year fixed effects and quarter fixed effects by year-by-quarter fixed effects. Specification 5 replaces quarter-of-year fixed effects by month-of-year fixed effects. Specification 6 adds indicators for swimming pool, gym and tennis court, and further adds (log) distance to the nearest shopping mall. Specification 7 additionally controls for buyer age; specifying a first-stage probit selection model and including the inverse mills ratio in the price regression yields very similar estimates. Standard errors, in parentheses, are clustered by building. Two-way clustering by building and purchase year yields similar values. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table A.4: Pairwise equality tests between models reported in Tables 2 and 4

Panel I: t-statistic of tests, Log Price A vs. Price B as the Dependent Variable

Both models	MSE Log Price vs. MSE Price	Equality test of models reported in:
r_t exponential	-5.595 (5-digit controls)	Table 2, column 1a vs. column 1b
r_t logarithmic	-5.554 (5-digit controls)	Table 2, column 3a vs. column 3b
r_t hyperbolic	-5.550 (5-digit controls)	Table 2, column 5a vs. column 5b
r_t exponential	-0.395 (3-digit controls)	Table 2, column 2a vs. column 2b
r_t logarithmic	-0.359 (3-digit controls)	Table 2, column 4a vs. column 4b
r_t hyperbolic	-0.355 (3-digit controls)	Table 2, column 6a vs. column 6b

Panel II: t-statistic of tests, Parametric Form A vs. Parametric Form B for r_t (dep.var. log price)

Model A	Model B	MSE Form A vs. MSE Form B
r_t exponential	r_t logarithmic	4.791 (5-digit controls)
r_t exponential	r_t hyperbolic	4.743 (5-digit controls)
r_t logarithmic	r_t hyperbolic	-1.814 (5-digit controls)
r_t exponential	r_t logarithmic	-5.519 (3-digit controls)
r_t exponential	r_t hyperbolic	-5.418 (3-digit controls)
r_t logarithmic	r_t hyperbolic	-2.221 (3-digit controls)

Panel III: t-statistic of tests, Parametric Form A vs. Step Function B for r_t (dep.var. log price)

Model A	MSE Parametric vs. MSE 5-step	Equality test of models reported in:
r_t exponential	5.027 (5-digit controls)	Table 2, column 1a vs. Table 4, column 3
r_t logarithmic	4.188 (5-digit controls)	Table 2, column 3a vs. Table 4, column 3
r_t hyperbolic	3.390 (5-digit controls)	Table 2, column 5a vs. Table 4, column 3
r_t exponential	-1.684 (3-digit controls)	Table 2, column 2a vs. Table 4, column 4
r_t logarithmic	5.070 (3-digit controls)	Table 2, column 4a vs. Table 4, column 4
r_t hyperbolic	4.906 (3-digit controls)	Table 2, column 6a vs. Table 4, column 4

Notes: This table reports on pairwise equality tests within and between the models reported in Tables 2 and 4. For each pairwise test, we show the t-statistic (1), defined in Appendix A.4, i.e., the difference between the Mean Square Error under model A and the Mean Square Error under model B, divided by the standard deviation of this difference. As such, a negative (resp., positive) t-statistic indicates that model A (resp., model B) exhibits the lower Mean Squared Error across out-of-sample observations. Models with 5-digit (resp., 3-digit) location controls include 178,588 (resp., 179,202) observations. See the referenced table and column for the Mean Squared Error under models A and B.

Table A.5: Sensitivity analysis: Jointly estimating the year-1 discount rate γ_1.

Function of time:	r_t exponential		r_t logarithmic		r_t hyperbolic	
Location controls:	5-digit	3-digit	5-digit	3-digit	5-digit	3-digit
	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable: Log price						
Year-1 rate, γ_1	0.0249	0.0334	0.0470	0.0655	0.0642	0.0836
	(0.0023)	(0.0032)	(0.0098)	(0.0059)	(0.0048)	(0.0006)
Slope parameter, γ_2	-0.0020	-0.0036	-0.0138	-0.0219	-0.2633	-0.3148
	(0.0008)	(0.0005)	(0.0051)	(0.0024)	(0.0133)	(0.0066)
Avg $\hat{\phi}_i$ 825-986y / avg $\hat{\phi}_i$ perp.	1.00	0.97	1.00	0.97	1.00	1.00
Avg $\hat{\phi}_i$ 87-99y / avg $\hat{\phi}_i$ perpetual	0.85	0.86	0.85	0.86	0.85	0.87
Model cross-valid. Mean Squared Error ($\times 10^6$ S\$/m^2 squared)	2.1184	6.8266	2.1184	6.8258	2.1184	6.8378

Notes: This table reports the estimated year-1 discount rate, γ_1, along with the slope parameter, γ_2, when the discount rate is a smooth parametric function of time, for 6 NLLS regressions. Standard errors, in parentheses, are clustered by building. The dependent variable is the log of the apartment's transaction price per square meter of floor area. Across columns, the discount rate is a smooth exponential, logarithmic, or hyperbolic function of time, and we further vary the granularity of location controls. Other housing utility shifters and market effects as in Table 2. All specifications use the full sample of 179,218 new apartment purchases from 1995 to 2015. Solver Knitro using the interior-point algorithm with the lower bound to the discount rate γ_3 fixed at 0.01% p.a., and without constraining the slope parameter γ_2. Instead of fixing the year-1 discount rate γ_1 at 4% p.a., we estimate this parameter subject to the constraint that it lies between 0.01% and 10%. For model cross validation (Appendix A.4), we randomly partition the data (within 5-digit or 3-digit location) into 10 folds; we take 9 folds as the training set and 1 fold as the test set, repeating ten times as we shift the test set to the next fold. We report the Mean Squared Error over observations in all 10 test sets.

Table A.6: Sensitivity analysis: A subsample of transactions in more established neighborhoods.

Location controls:	5-digit		3-digit	
Sample composition:	Full sample: All areas	Areas w/ longer lease types	Full sample: All areas	Areas w/ longer lease types
	(1)	(2)	(3)	(4)
Dependent variable: Log price				
	Function of time: r_t exponential			
Slope parameter, γ_2	-0.0046	-0.0049	-0.0046	-0.0046
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
	Function of time: r_t logarithmic			
Slope parameter, γ_2	-0.0102	-0.0123	-0.0097	-0.0104
	(0.0005)	(0.0002)	(0.0004)	(0.0005)
	Function of time: r_t hyperbolic			
Slope parameter, γ_2	-0.1537	-0.2173	-0.1430	-0.1591
	(0.0119)	(0.0150)	(0.0078)	(0.0092)

Notes: This table reports the slope parameter, γ_2, when the discount rate is a smooth parametric function of time, for 12 NLLS regressions. Standard errors, in parentheses, are clustered by building. The dependent variable is the log of the apartment's transaction price per square meter of floor area. Specifications in columns 1 and 3 use the full sample of 179,218 transactions across areas. Specifications in columns 2 and 4 use the subsample of 31,072 transactions in areas in which at least multi-century and perpetual leases were traded in sample. As indicated from left to right, we vary the granularity of location controls, from 5 digit to 3 digit. Other housing utility shifters and market effects as in Table 2. As indicated from top to bottom, the discount rate is an exponential, logarithmic, or hyperbolic function of time. Solver Knitro using the interior-point algorithm with γ_1 and γ_3 fixed at 4% and 0.01% p.a., respectively.

Table A.7: Sensitivity analysis: Time-unlimited ownership can shift utility.

Function of time:	r_t exponential	r_t logarithmic	r_t hyperbolic
Dependent variable: Log price	(1a)	(2a)	(3a)
Slope parameter, γ_2	-0.0055	-0.0086	-0.1206
	(0.0004)	(0.0008)	(0.0136)
"Ownership forever" (perpetual lease, 1=yes)	-0.1718	0.0348	0.0349
	(0.1131)	(0.0148)	(0.0143)
Dependent var.: Price per m^2	(1b)	(2b)	(3b)
Slope parameter, γ_2	-0.0041	-0.0073	-0.0974
	(0.0018)	(0.0039)	(0.0664)
"Ownership forever" (perpetual lease, 1=yes)	0.0193	0.0213	0.0210
	(0.0410)	(0.0142)	(0.0145)

Notes: This table shows sensitivity to including a perpetual lease dummy ("ownership forever") in the set of property characteristics of the utility specification, thus reporting its estimated coefficient $\theta_{forever}$, along with the slope parameter γ_2 of the discount rate schedule. Standard errors, in parentheses, are clustered by building. Across columns, the discount rate is a smooth exponential, logarithmic, or hyperbolic function of time. In the top panel (columns 1a-3a), the dependent variable is the log of the apartment's transaction price per square meter of floor area. In the bottom panel (columns 1b-3b), the dependent variable is the apartment's transaction price per square meter of floor area (S$/m^2). All 6 NLLS regressions are implemented on the full sample of 179,218 new apartment purchases, and specify location controls at the 3-digit level (similar patterns obtain with 5-digit location controls). Besides ownership forever and location controls, housing utility shifters and market effects as in Table 2. Solver Knitro using the interior-point algorithm with γ_1 and γ_3 fixed at 4% and 0.01% p.a., respectively.

Table A.8: Sensitivity analysis: Varying the year-1 discount rate γ_1.

Function of time:	r_t exponential (1)	r_t logarithmic (2)	r_t hyperbolic (3)
Dependent variable: Log price			
	γ_1 fixed at 3% p.a. (lower than baseline)		
Slope parameter, γ_2	-0.0031	-0.0045	-0.0765
	(0.0001)	(0.0004)	(0.0081)
	γ_1 fixed at 4% p.a. (Table 2)		
Slope parameter, γ_2	-0.0046	-0.0097	-0.1430
	(0.0001)	(0.0004)	(0.0078)
	γ_1 fixed at 5% p.a. (higher than baseline)		
Slope parameter, γ_2	-0.0058	-0.0149	-0.1948
	(0.0001)	(0.0004)	(0.0072)
	γ_1 fixed at 6% p.a. (higher than baseline)		
Slope parameter, γ_2	-0.0069	-0.0196	-0.2373
	(0.0001)	(0.0001)	(0.0072)

Notes: This table shows sensitivity to fixing the year-1 discount rate γ_1 at different levels, as indicated from top to bottom. Standard errors, in parentheses, are clustered by building. Across columns, the discount rate is a smooth exponential, logarithmic, or hyperbolic function of time. The dependent variable is the log of the apartment's transaction price per square meter of floor area. All 12 NLLS regressions are implemented on the full sample of 179,218 new apartment purchases, and specify location controls at the 3-digit level (similar patterns obtain with 5-digit controls). Other housing utility shifters and market effects as in Table 2. Solver Knitro using the interior-point algorithm with the lower bound to the discount rate γ_3 fixed at 0.01% p.a.

Table A.9: Sensitivity analysis: Varying the lower bound to the discount rate γ_3.

Function of time:	r_t exponential (1)	r_t logarithmic (2)	r_t hyperbolic (3)
Dependent variable: Log price			
	γ_3 fixed at 0.001% p.a. (lower than baseline)		
Slope parameter, γ_2	-0.0037	-0.0097	-0.1430
	(0.0001)	(0.0004)	(0.0078)
	γ_3 fixed at 0.01% p.a. (Table 2)		
Slope parameter, γ_2	-0.0046	-0.0097	-0.1430
	(0.0001)	(0.0004)	(0.0078)
	γ_3 fixed at 0.1% p.a. (higher than baseline)		
Slope parameter, γ_2	-0.0056	-0.0097	-0.1430
	(0.0002)	(0.0004)	(0.0078)

Notes: This table shows sensitivity to fixing the lower bound to the discount rate γ_3 at different levels, as indicated from top to bottom. Standard errors, in parentheses, are clustered by building. Across columns, the discount rate is a smooth exponential, logarithmic, or hyperbolic function of time. The dependent variable is the log of the apartment's transaction price per square meter of floor area. All 9 NLLS regressions are implemented on the full sample of 179,218 new apartment purchases, and specify location controls at the 3-digit level (similar patterns obtain with 5-digit controls). Other housing utility shifters and market effects as in Table 2. Solver Knitro using the interior-point algorithm with the year-1 discount rate γ_1 fixed at 4% p.a.

Table A.10: Sensitivity analysis: Discount rate varying with buyer age (where available).

Function of time:	r constant		r_t exponential		r_t logarithmic		r_t hyperbolic	
Location controls:	5-digit (1)	3-digit (2)	5-digit (3)	3-digit (4)	5-digit (5)	3-digit (6)	5-digit (7)	3-digit (8)
Dependent variable: Log price								
Discount rate, r_{young} (p.a.)	0.0205	0.0212						
	(0.0011)	(0.0009)						
Discount rate, r_{old} (p.a.)	0.0208	0.0216						
	(0.0011)	(0.0009)						
Slope parameter, $\gamma_{2,young}$			-0.0046	-0.0045	-0.0102	-0.0097	-0.1530	-0.1433
			(0.0001)	(0.0001)	(0.0005)	(0.0004)	(0.0078)	(0.0067)
Slope parameter, $\gamma_{2,old}$			-0.0047	-0.0046	-0.0102	-0.0099	-0.1543	-0.1471
			(0.0001)	(0.0001)	(0.0005)	(0.0004)	(0.0078)	(0.0068)
Avg $\hat{\phi}_i$ 825-986y / avg $\hat{\phi}_i$ perp.	1.00	1.00	0.93	0.95	1.00	1.00	1.00	1.00
Avg $\hat{\phi}_i$ 87-99y / avg $\hat{\phi}_i$ perpetual	0.86	0.87	0.85	0.87	0.85	0.86	0.85	0.86

Notes: This table shows sensitivity to allowing the discount rate schedule to vary according to buyer age. Standard errors, in parentheses, are clustered by building. The dependent variable is the log of the apartment's transaction price per square meter of floor area. Across columns, the discount rate is a constant, exponential, logarithmic, or hyperbolic function of time, and we further vary the granularity of location controls. Other housing utility shifters and market effects as in Table 2. All 8 NLLS regressions are implemented on the subsample of 54,548 new apartment purchases from 1995 to 2015 for which buyer age is observed. We allow the discount rate schedule to shift if buyer age exceeds the median of 41 years in this subsample, i.e., the constant rate is r_{old} if buyer age exceeds the median and r_{young} otherwise, or the slope parameter is $\gamma_{2,old}$ if buyer age exceeds the median and $\gamma_{2,young}$ otherwise. For parametric structures, we fix the year-1 discount rate γ_1 at 4% p.a. and the lower bound to the discount rate γ_3 at 0.01% p.a.

Table A.11: Sensitivity analysis: Varying the rate acceleration penalty λ (5-step schedule).

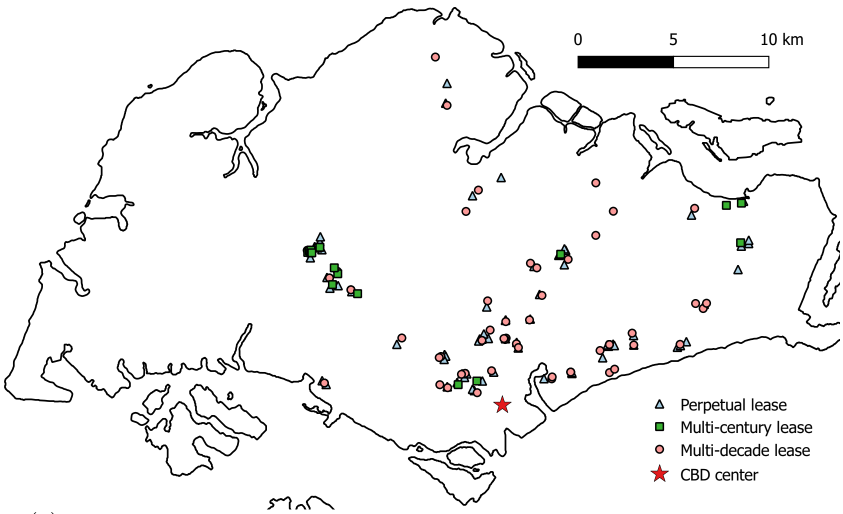
Location controls:	5-digit			3-digit		
Smoothing parameter, λ:	1514 (half) (1)	3027 (Table 4) (2)	6054 (double) (3)	1514 (half) (4)	3027 (Table 4) (5)	6054 (double) (6)
Dependent variable: Log price						
Discount rate, year 1 to 100 (p.a.)	0.0253	0.0247	0.0239	0.0327	0.0320	0.0311
	(0.0008)	(0.0008)	(0.0009)	(0.0001)	(0.0001)	(0.0001)
Discount rate, year 101 to 200 (p.a.)	0.0167	0.0172	0.0175	0.0201	0.0196	0.0195
	(0.0008)	(0.0008)	(0.0009)	(0.0001)	(0.0001)	(0.0001)
Discount rate, year 201 to 300 (p.a.)	0.0106	0.0114	0.0124	0.0069	0.0081	0.0092
	(0.0008)	(0.0008)	(0.0009)	(0.0001)	(0.0001)	(0.0001)
Discount rate, year 301 to 400 (p.a.)	0.0072	0.0077	0.0089	0.0001	0.0017	0.0029
	(0.0008)	(0.0008)	(0.0009)	(0.0001)	(0.0001)	(0.0001)
Discount rate, year 401 on (p.a.)	0.0059	0.0059	0.0072	0.0013	0.0012	0.0010
	(0.0008)	(0.0008)	(0.0009)	(0.0001)	(0.0001)	(0.0001)
Average $\hat{\phi}_i$ 825-986y/avg. $\hat{\phi}_i$ perp.	1.00	1.00	1.00	0.97	0.97	0.97
Average $\hat{\phi}_i$ 87-99y / avg. $\hat{\phi}_i$ perp.	0.85	0.85	0.85	0.86	0.86	0.86
Obj. function components (fitted):						
Sum of squared second diff. (SSSD)	0.000020	0.000014	0.000010	0.000108	0.000061	0.000041
Rate acceleration penalty, $\lambda\times$SSSD	0.0299	0.0438	0.0627	0.1632	0.1856	0.2477
Residual sum of squares (RSS)	1513.42	1513.43	1513.45	3713.32	3713.41	3713.50
Objective = RSS+$\lambda\times$SSSD	1513.45	1513.47	1513.51	3713.48	3713.60	3713.75
Model cross-valid. Mean Squared Error ($\times 10^6$ S\$/m^2 squared)	2.1182	2.1183	2.1183	6.8219	6.8225	6.8232

Notes: This table shows sensitivity of discount rates as a step function of time (5 steps) when the specified smoothing weight λ varies around 3027 (per Table 4). Standard errors, in parentheses, are clustered by building. The dependent variable is the log of the apartment's transaction price per square meter of floor area. We further vary the granularity of location controls, as indicated. Other housing utility shifters and market effects as in Table 2. All specifications use the full sample of 179,218 new apartment purchases from 1995 to 2015. Solver Knitro using the interior-point algorithm with discount rates r_1 to r_5 constrained between 0 and 10% p.a. (i.e., 0.1). The objective function adds to the RSS a rate acceleration penalty given by a smoothing parameter λ times the sum of squared second differences (SSSD), i.e., $((r_3 - r_2) - (r_2 - r_1))^2 + (r_4 - r_3) - (r_3 - r_2))^2 + ...$ where $r_1, r_2, r_3, ...$ denote annual discount rates from year 1 to 100, year 101 to 200, year 201 to 300,... As shown, the rate acceleration penalty accounts for a small fraction of the optimized objective function, e.g., 0.04 out of 1513.47 in column 2. For model cross validation (Appendix A.4), we randomly partition the data (within 5-digit or 3-digit location) into 10 folds; we take 9 folds as the training set and 1 fold as the test set, repeating ten times as we shift the test set to the next fold. We report the Mean Squared Error over observations in all 10 test sets.

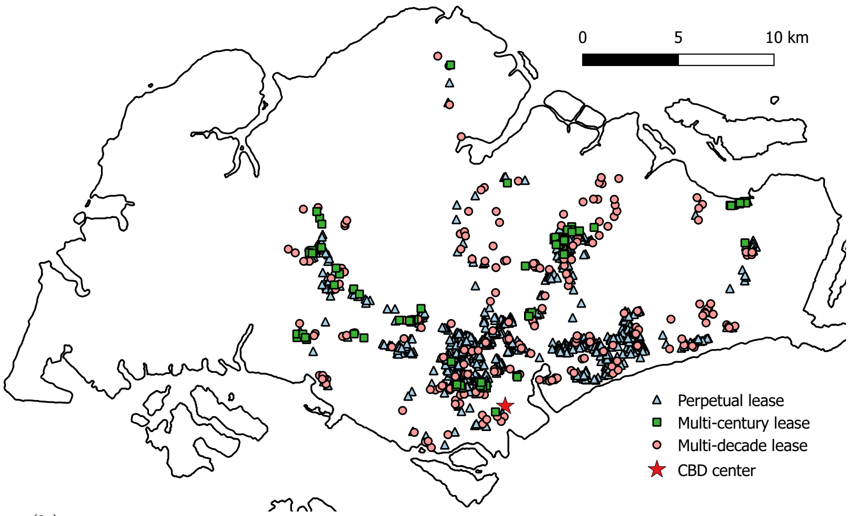
Table A.12: Monthly rental price does not vary with the landlord's lease.

Dependent variable: Log rental price						
Location controls:	5-digit	4-digit	3-digit	5-digit	4-digit	3-digit
	(1)	(2)	(3)	(4)	(5)	(6)
Landlord lease 825 to 986 years (1=yes)		0.005	-0.010		0.003	-0.002
		(0.029)	(0.017)		(0.030)	(0.018)
Landlord lease 87 to 99 years (1=yes)	-0.036*	0.033	0.017	-0.035	0.026	0.002
	(0.020)	(0.021)	(0.016)	(0.024)	(0.021)	(0.015)
5-digit location fixed effects	Yes			Yes		
4-digit location fixed effects		Yes			Yes	
3-digit location fixed effects			Yes			Yes
Distance to subway (in log)				Yes	Yes	Yes
R^2	0.926	0.915	0.892	0.926	0.915	0.895
Number of regressors	731	465	201	732	466	202

Notes: This table shows estimates for 6 OLS regressions. The dependent variable is the log of the apartment's monthly rental price per square meter of floor area (S$ base CPI January 2014). The sample consists of 42,699 rental contracts between February 2012 and January 2015 for privately developed apartments aged less than ten years located across all areas; we also drop one-bedroom contracts (which include the renting out of a single bedroom) and trim the top and bottom 1% of the rental price distribution. In all specifications, the dummy for perpetual landlord leases is the reference category. We vary the granularity of location controls, from 5 digit to 3 digit, as indicated. The rental data report the apartment's floor area in ranges (300-400, 400-500, ..., 2900-3000, above 3000 ft^2) and the number of bedrooms (one, two, three, four, five and above). The rental data do not report the apartment's story, and we roughly proxy for this using bins for the maximal story across buildings in the project (1-4, 5-8, 9-12, 13-19, 20-29, 30-39, 40-49, 50-59, 60-70). Besides location, all specifications include floor-area bins; number-of-bedroom bins; maximal-story-in-project bins; project size (in log); a project-specific indicator for a large land parcel; age bins of width 1 year; year-of-contract fixed effects; and quarter-of-year fixed effects. Relative to specifications 1-3, specifications 4-6 control for distance to the nearest subway (Mass Rapid Transit, MRT) station. Standard errors, in parentheses, are two-way clustered by project and contract year. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.



(a) Condominiums with 5-digit postal codes containing at least two lease types



(b) Condominiums with 3-digit postal codes containing at least two lease types

Figure A.1: Tight location controls remove variation in lease length. Condominium project locations for leases that are not perfectly predicted by (a) 5-digit controls, and (b) 3-digit controls. For example, the 5-digit code 12772x perfectly predicts Botannia's "956-years-from-1928" lease, so this condominium project is not illustrated in (a); by contrast, the 3-digit code 127xxx also contains Infiniti apartments on perpetual leases, so both the Botannia and the Infiniti condominium projects are illustrated in (b). CBD marks the Central Business District, centered at One Raffles Place.

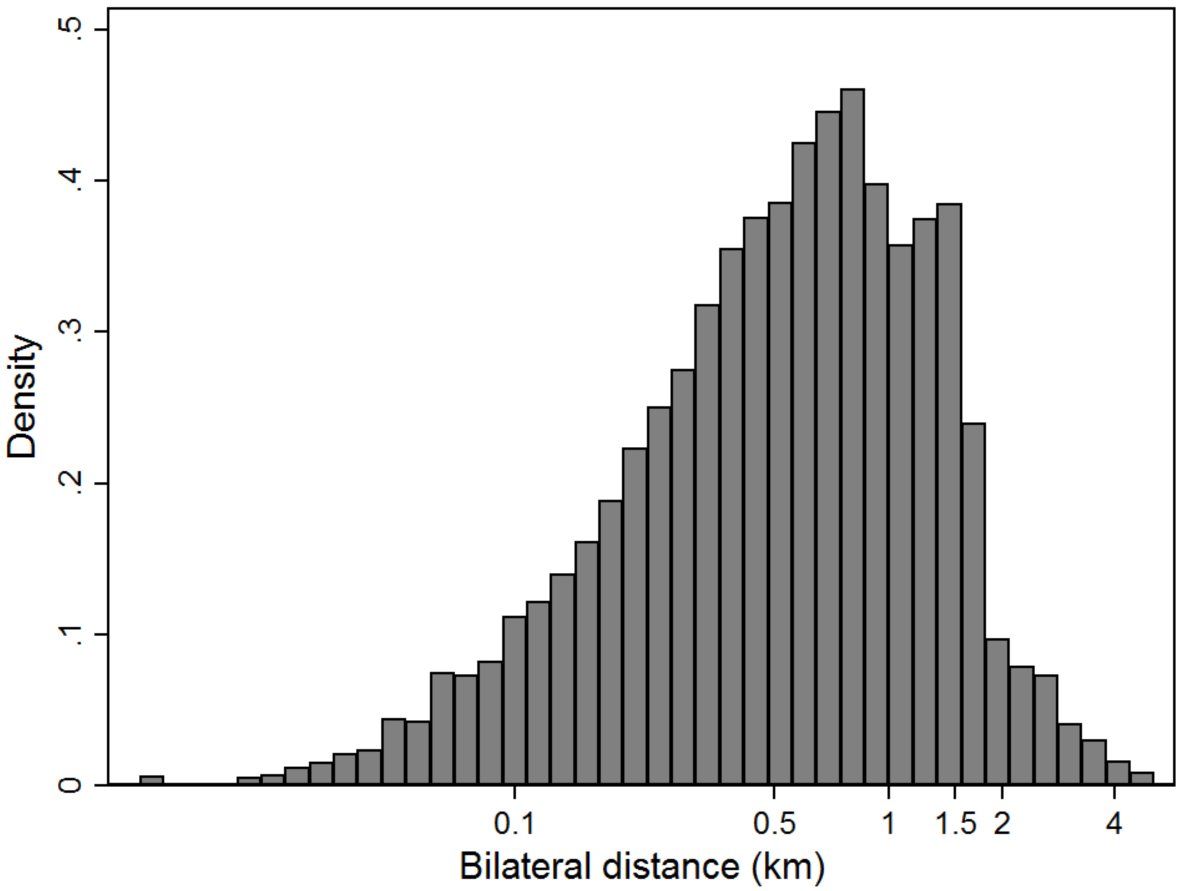
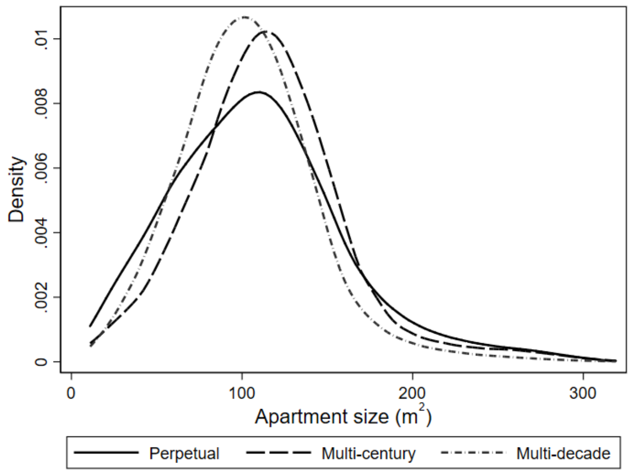
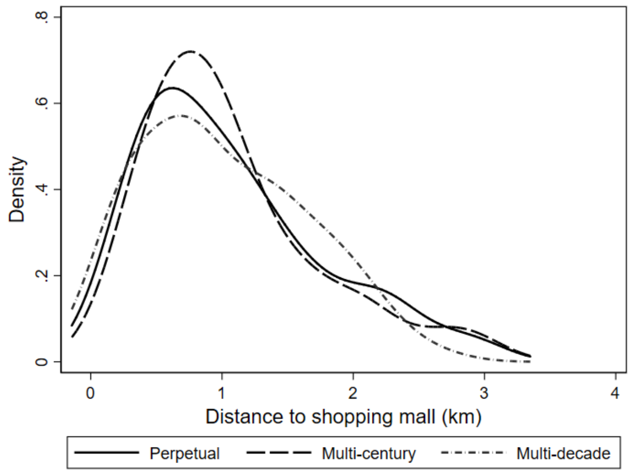


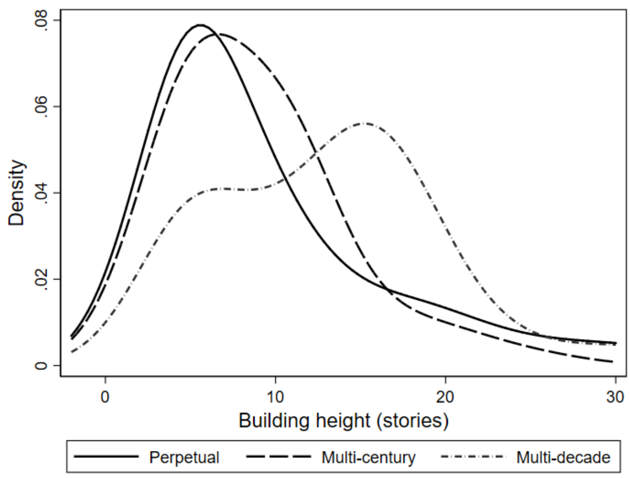
Figure A.2: Bilateral distance between condominium project pairs within the same 3-digit location in the sample (as in the multi-century-lease Botannia and perpetual-lease Infiniti example, the two neighboring projects located at 127xxx). The distribution shows that while 3-digit locations (187 locations in all) are less narrowly defined than 5-digit locations (1516 locations in all), they are still quite granular.



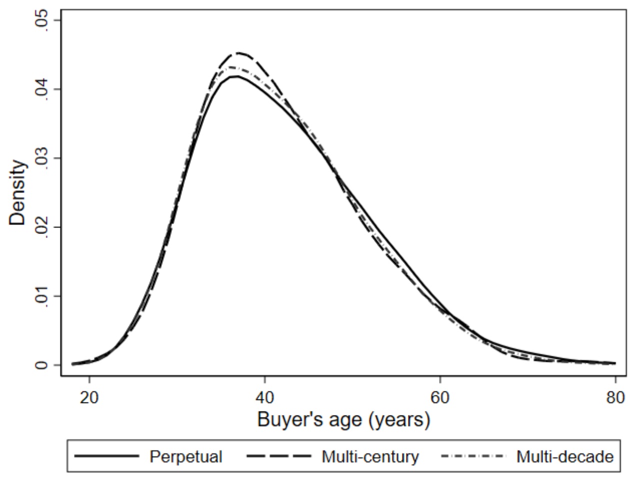
(a) Apartment size, by lease type



(b) Distance to mall, by lease type

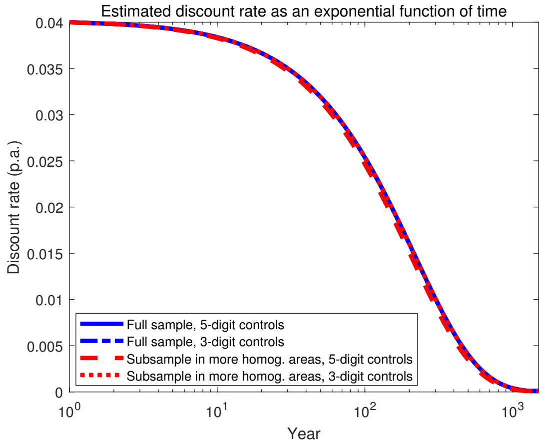


(c) Building height, by lease type

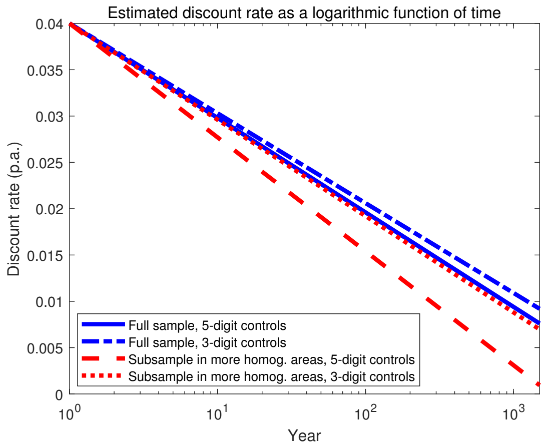


(d) Buyer age, by lease type

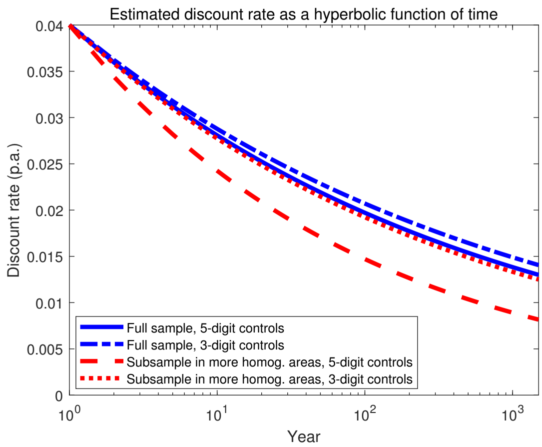
Figure A.3: Distributions by lease type over (a) apartment size, (b) apartment's distance to the nearest shopping mall (in 2015, as a further proxy for fine neighborhood characteristics), (c) apartment building height, (d) and buyer age. An observation is: (a) a new apartment sold in the 1995-2015 period; (b), (c) an apartment building recording sales in the 1995-2015 period; and (d) a new apartment sold in the 1995-2012 period for which buyer age is observed (He et al., 2020).



(a) r_t as an exponential function of time

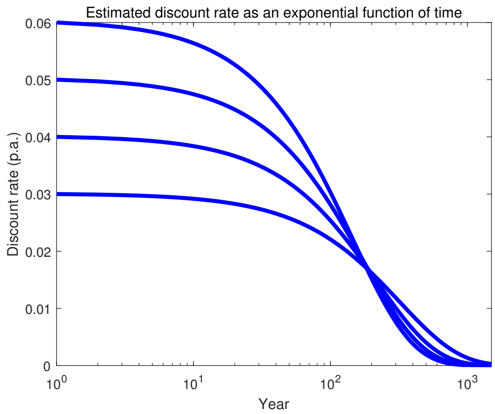


(b) r_t as a logarithmic function of time

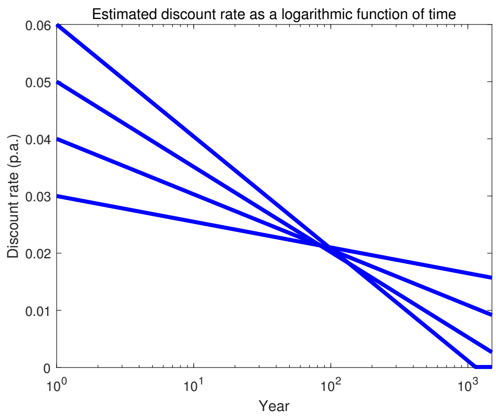


(c) r_t as a hyperbolic function of time

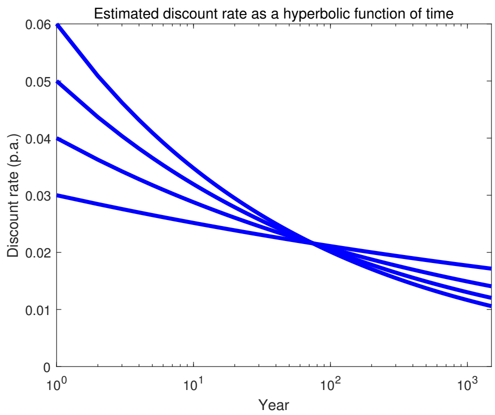
Figure A.4: Sensitivity of discount rates as the sample composition varies (within panel), for different parametric forms over time: (a) exponential, (b) logarithmic, and (c) hyperbolic, plotted on a log time scale. The discount rate r_t discounts benefits from year $t+1$ to year t. Source: Table A.6 estimates based on the subsample of 31,072 transactions in more established areas in which at least multi-century and perpetual leases were traded in sample, besides estimates based on the full sample of 179,218 transactions across areas. We show estimates with 5-digit or with 3-digit location controls.



(a) r_t as an exponential function of time



(b) r_t as a logarithmic function of time



(c) r_t as a hyperbolic function of time

Figure A.5: Sensitivity of discount rates as the year-1 rate varies, for different parametric forms over time: (a) exponential, (b) logarithmic, and (c) hyperbolic, plotted on a log time scale. The discount rate r_t discounts benefits from year $t+1$ to year t. Source: Table A.8 estimates based on the full sample, varying the year-1 discount rate γ_1 (within panel), and fixing location controls at the 3-digit level (similar patterns obtain with 5-digit controls) and the lower-bound rate γ_3 at 0.01% p.a.

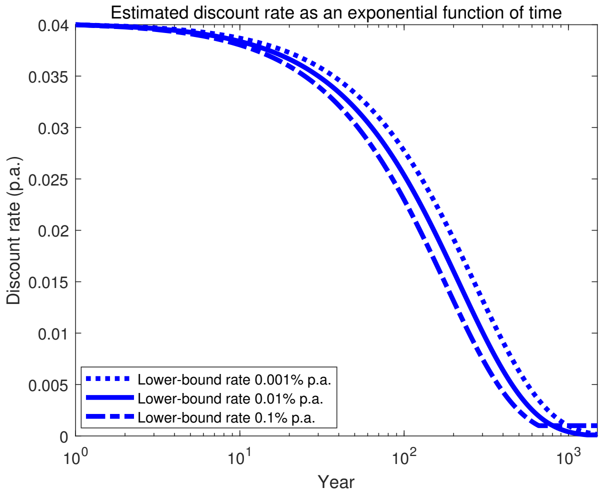
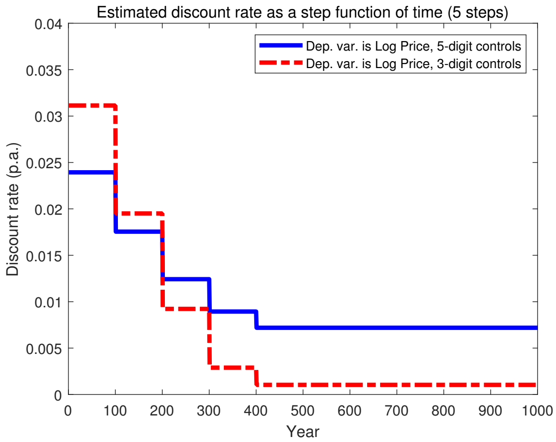
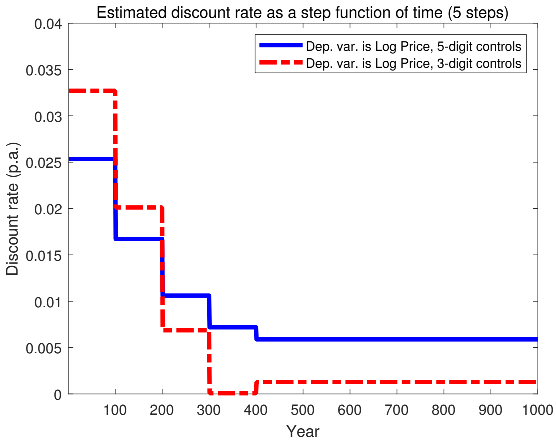


Figure A.6: Sensitivity of discount rates as the lower bound to the discount rate varies, plotted on a log time scale. Exponential form over time (no change for the other parametric forms). The discount rate r_t discounts benefits from year $t+1$ to year t. Source: Table A.9 estimates based on the full sample, varying the lower-bound rate γ_3 (within panel), and fixing location controls at the 3-digit level (similar patterns obtain with 5-digit location controls) and the year-1 rate γ_1 fixed at 4% p.a.

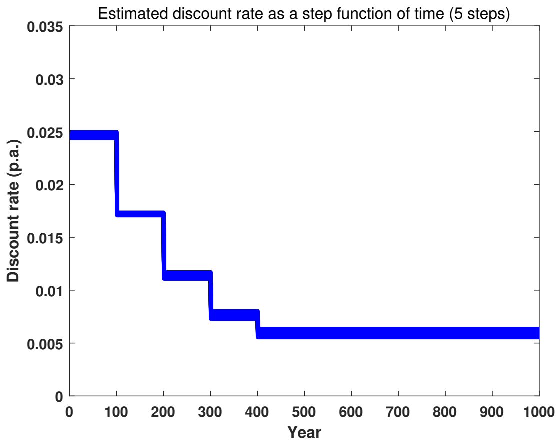


(a) r_t with double the acceleration penalty

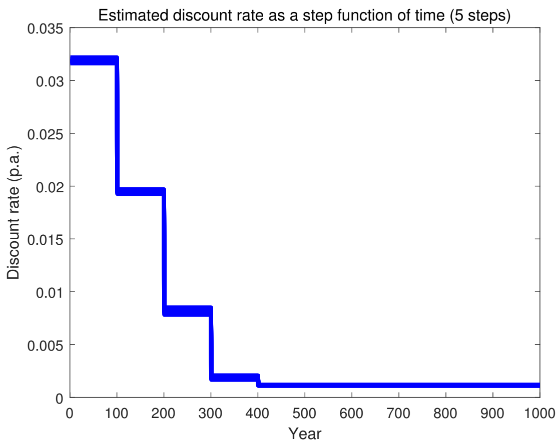


(b) r_t with half the acceleration penalty

Figure A.7: Sensitivity of discount rates as a step function of time when the specified smoothing weight λ is (a) doubled to 6054 or (b) halved to 1514, plotted on a linear time scale. This figure should be compared to Figure 4, panel (b), based on $\lambda = 3027$ (5-step schedule from Table 4, columns 3-4). Source: Table A.11 estimates based on the full sample, varying the location fixed effects (within panel).



(a) r_t estimated by fold (5-step with 5-digit controls)



(b) r_t estimated by fold (5-step with 3-digit controls)

Figure A.8: Stability of discount rate schedule parameter estimates when we randomly partition the data (within 5-digit or 3-digit location) into 10 folds and take 9 folds as the training set. See Appendix A.4. Each panel superimposes 10 sets of estimates, one for each of 10 repetitions (training sets). Specifications follow that of Table 4, columns 3-4 exactly, i.e., five steps one century wide until the fifth century, with (a) 5-digit location fixed effects or (b) 3-digit fixed effects. Besides this illustration, parameter estimates are stable across training sets for all specifications reported in the paper.