A Robust Approach to Estimating Production Functions: Replication of the ACF Procedure

Not-for-publication Online Appendix

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Abstract

This not-for-publication online appendix discusses why including "1" in the instrument of our modified moment condition helps with identification in the ACF Monte Carlo setup.

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Consider a Leontief production function as in ACF

$$Y_{it} = \min\left\{e^{\beta_0} K_{it}^{\beta_k} L_{it}^{\beta_l} e^{\omega_{it}}, e^{\beta_m} M_{it}\right\} e^{\eta_{it}}.$$

In this Leontief production function setting, we have

$$\beta_m + m_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \omega_{it}, \tag{1}$$

and hence, substituting ω_{it} — the inverse intermediate input demand function — into the value-added production function, we obtain

$$\begin{aligned} y_{it} &= \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \eta_{it} \\ &= \beta_0 + \beta_k k_{it} + \beta_l l_{it} + [\beta_m + m_{it} - \beta_0 - \beta_k k_{it} - \beta_l l_{it}] + \eta_{it} \\ &= \beta_m + m_{it} + \eta_{it}. \end{aligned}$$

This implies the population equation $\widetilde{\Phi}_t(l_{it}, k_{it}, m_{it})$ of ACF's first stage becomes

$$\widetilde{\Phi}_t(l_{it}, k_{it}, m_{it}) = \beta_m + m_{it}.$$

It follows that

$$\widetilde{\Phi}_t(l_{it}, k_{it}, m_{it}) - \beta_k k_{it} - \beta_l l_{it} = \beta_0 + \omega_{it}. \tag{2}$$

When we use $\widehat{\beta_0 + \omega_{it}} = \widehat{\widetilde{\Phi}}_t(l_{it}, k_{it}, m_{it}) - \beta_k k_{it} - \beta_l l_{it}$ for the autoregressive regression of productivity in place of ω_{it} as in the concentrated ACF procedure, we should include the intercept as

$$(\widehat{\beta_0 + \omega_{it}}) = \alpha_0 + \rho(\widehat{\beta_0 + \omega_{i,t-1}}) + \xi_{it}, \tag{3}$$

because the above regression becomes equivalent to $\widehat{\omega_{it}} = \rho\widehat{\omega_{i,t-1}} + \xi_{it}$ only with $\alpha_0 = \beta_0(1-\rho)$. This also makes residual $\xi_{it}(\beta_l, \beta_k)$ have mean zero by construction, regardless of whether the intercept is actually equal to the true $\alpha_0 = \beta_0(1-\rho)$.

Below, we argue that identification is improved if we remove this constant term, or equivalently, if we run the AR(1) regression with productivity only. To make this argument, we first need to see how a spurious identification point arises in the original ACF moment condition. Let the spurious minimum be $\tilde{\beta}_k = 0$ and $\tilde{\beta}_l = \beta_l + \beta_k = 1$. Also, write $\widetilde{\Phi}_{it} = \widetilde{\Phi}_t(l_{it}, k_{it}, m_{it})$. From (2), we then obtain

$$\widetilde{\Phi}_{it} - \tilde{\beta}_k k_{it} - \tilde{\beta}_l l_{it} = \beta_0 + \beta_k k_{it} - (1-\beta_l) l_{it} + \omega_{it}. \tag{4}$$

Also, note that from the optimal labor input in the ACF DGP, we have $(1-\beta_l)l_{it}$ as

$$(1-\beta_l)l_{it} = \beta_0 + \ln\beta_l - \ln W_{it} + \beta_k k_{it} + (\rho^b\omega_{i,t-b} + \frac{1}{2}\sigma^2). \tag{5}$$

where W_{it} denotes the wage and the productivity $\omega_{i,t-1}$ evolves to $\omega_{i,t-b}$, at which point in time the firm chooses labor input as in DGP1 (see their equation (37)).

Plugging (5) into (4), we obtain

$$\widetilde{\Phi}_{it} - \tilde{\beta}_k k_{it} - \tilde{\beta}_l l_{it} = -\ln\beta_l - \sigma^2/2 + \ln W_{it} - \rho^b\omega_{i,t-b} + \omega_{it} = (-\ln\beta_l - \sigma^2/2) + \xi_{it}^B + \ln W_{it}. \tag{6}$$

because $\omega_{it} = \rho^b\omega_{i,t-b} + \xi_{it}^B$. Equation (6) has two important implications. First, the constant term $(-\ln\beta_l - \sigma^2/2)$ is different from β_0 in (2). Second, the regression residual from the AR(1) regression at the spurious minimum using $\widetilde{\Phi}_{it} - \tilde{\beta}_k k_{it} - \tilde{\beta}_l l_{it}$ is also different from ξ_{it} in (3). Note that the wage follows an AR(1) process as $\ln W_{it} = \rho_W \ln W_{i,t-1} + \xi_{it}^W$ and its innovation term ξ_{it}^W is independent of $(k_{it}, l_{i,t-1})$. Therefore, the spurious parameter $(\tilde{\beta}_k = 0, \tilde{\beta}_l = 1)$ solves the ACF's moment condition as well as the true parameter, as discussed in ACF's footnote 16.

Note that in the ACF DGP, we have $\beta_0 = 0$. Therefore, running the AR(1) regression without the intercept helps the moment condition to yield a "true" solution, deterring the spurious minimum because the spurious minimum solution requires the AR(1) regression to have a non-zero intercept (or an intercept different from $\alpha_0 = \beta_0(1-\rho)$). Without the intercept, the "spurious" innovation term $\xi_{it}^B - \rho_W\xi_{it}^B + \xi_{it}^W$ in the "spurious" AR(1) regression using (6) would not have mean zero, which is inconsistent with the true DGP.

In our Monte Carlos experiment, the AR(1) regression does not include the intercept, i.e. we estimate $\widehat{\omega_{it}} = \rho\widehat{\omega_{i,t-1}} + \xi_{it}$ to obtain the residual. Regressing without an intercept deters ξ_{it} from having a zero mean when the parameter values differ from that of the true ones. Instead, we include "1" in the instrument so that we can ensure innovation ξ_{it} has mean zero at the true parameter values. This helps for identification away from the spurious minimum. If we included a constant in the regression, the residual would have zero mean for any parameter values that include the spurious solution.

In general this constant β_0 in the production function is not known. Moreover, it is not separately identified from the mean of the unobserved productivity. However, the sum of β_0 and the mean of the productivity can be estimated in the first stage of the ACF procedure. From (2), note that this sum, whether it is zero or not, is obtained as the constant term of the function $\widetilde{\Phi}_t(l_{it}, k_{it}, m_{it})$. Note that this sum is equal to β_m in the Leontief production function (see equation (1)), which can be easily estimated using OLS of y_{it} on m_{it} since $y_{it} = \beta_m + m_{it} + \eta_{it}$. After removing this constant from $\widehat{\beta_0 + \omega_{it}}$, we can then use the regression $\widehat{\omega_{it}} = \rho\widehat{\omega_{i,t-1}} + \xi_{it}$ to obtain the innovation term, where $\widehat{\omega_{it}}$ becomes a kind of de-meaned productivity if the productivity does not have mean zero.