Corrigendum to: Assessing International Commonality in Macroeconomic Uncertainty and Its Effects*

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Abstract

Carriero, Clark, and Marcellino (2020, CCM2020) used large BVAR models with a factor structure to stochastic volatility to produce estimates of time-varying international macroeconomic uncertainty and assess uncertainty's effects on the global economy. The results in CCM2020 were based on an estimation algorithm that has recently been shown to be incorrect by Bognanni (2021) and fixed by Carriero, et al. (2021). In this note we use the algorithm correction of Carriero, et al. (2021) to correct the estimates of CCM2020. Although the correction has some impact on the original results, the changes are small and the key findings of CCM2020 are upheld.

Keywords: Triangular algorithm, business cycle uncertainty, stochastic volatility

J.E.L. Classification: E32, E44, C11, C55

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1 Introduction

Carriero, Clark, and Marcellino (2020, hereafter denoted CCM2020) used large BVAR models with a particular volatility structure to study the cross-country commonality of uncertainty and its effects. To make tractable the estimation of the required large model, CCM2020 used an equation-by-equation approach to the vector autoregression (VAR) based on a triangularization of the conditional posterior distribution of the coefficient vector developed in Carriero, Clark, and Marcellino (2019, hereafter CCM2019). However, Bognanni (2021) recently identified a conceptual problem with the triangular algorithm of CCM2019; the triangularization does not deliver the intended posterior of the VAR's coefficients. The same problem afflicts the estimation algorithm used in CCM2020.

In response, Carriero, et al. (2021) have developed a corrected triangular algorithm for Bayesian VARs that does yield the intended posterior. This new algorithm permits an equation-by-equation approach to the VAR and offers the same basic computational advantages of the original triangular algorithm. In addition, the new algorithm can be used to properly estimate the uncertainty model of CCM2020.

In this note, we provide corrected versions of the published results of CCM2020. Drawing from Carriero, et al. (2021), Section 2 briefly explains the problem with the original triangular algorithm and the correction. Section 3 presents corrected versions of the results of CCM2020. Although the correction has some impact on results, these impacts are small, and the key findings of CCM2020 are upheld.

2 Original algorithm and correction

For convenience, we briefly detail the models used in CCM2020: first the one-factor BVAR-GFSV model applied to the 19-country GDP data set and then the two-factor model applied to the three-economy macroeconomic data set. In the interest of brevity, we do not spell out the simpler BVAR with stochastic volatility (BVAR-SV) used for some results in CCM2020; the results below include some updates of these results that are also affected by correcting the equation-by-equation algorithm the paper used to estimate the BVAR-SV models.

2.1 One-factor BVAR-GFSV model

Let y_t denote the $n \times 1$ vector of variables of interest, covering multiple countries. The $n \times 1$ vector of reduced-form shocks to these variables is:

$$v_t = A^{-1}\Lambda_t^{0.5}\epsilon_t,\ \epsilon_t \sim iid\ N(0, I), \tag{1}$$

where A is an $n \times n$ lower triangular matrix with ones on the main diagonal, and Λ_t is a diagonal matrix of volatilities, $\lambda_{i,t}, i = 1, \ldots, n$. For each variable i, its log-volatility follows a linear factor model with a common uncertainty factor $\ln m_t$ that follows an AR(p_m) process augmented to include y_{t-1} and an idiosyncratic component $\ln h_{i,t}$ that follows an AR(1) process:

$$\begin{aligned}
\ln \lambda_{i,t} &= \beta_{m,i} \ln m_t + \ln h_{i,t},\ i = 1, \ldots, n & (2)\\
\ln m_t &= \sum_{i=1}^{p_m} \delta_{m,i} \ln m_{t-i} + \delta'_{m,y} y_{t-1} + u_{m,t},\ u_{m,t} \sim iid\ N(0, \phi_m) & (3)\\
\ln h_{i,t} &= \gamma_{i,0} + \gamma_{i,1} \ln h_{i,t-1} + e_{i,t},\ i = 1, \ldots, n. & (4)
\end{aligned}$$

The volatility factor m_t is our measure of (unobservable) *global* macroeconomic uncertainty. The idiosyncratic component $h_{i,t}$ captures time variation in a country's GDP volatility unique to that country. The uncertainty shock $u_{m,t}$ is independent of the conditional errors ϵ_t and the vector of volatility innovations $\nu_t = (e_{1,t}, \ldots, e_{n,t})'$, which is jointly distributed as iid $N(0, \Phi_\nu)$ with elements independent among themselves, so that $\Phi_\nu = diag(\phi_1, \ldots, \phi_n)$. For identification, we follow common practice in the dynamic factor model literature and assume $\ln m_t$ to have a zero unconditional mean, fix the variance ϕ_m at 0.03, and use a simple accept-reject step to restrict the first variable's (US GDP growth) loading to be positive.

The global uncertainty measure m_t can also affect the levels of the macroeconomic variables contained in y_t, contemporaneously and with lags. In particular, y_t is assumed to follow:

$$y_t = \sum_{i=1}^{p} \Pi_i y_{t-i} + \sum_{i=0}^{p_m} \Pi_{m,i} \ln m_{t-i} + v_t, \tag{5}$$

where p denotes the number of y_t lags in the VAR, p_m denotes the number of $\ln m_t$ lags in the conditional mean of the VAR (for computational convenience, set to the lag order of the factor process), Π_i is an $n \times n$ matrix, $i = 1, \ldots, p$, and $\Pi_{m,i}$ is an $n \times 1$ vector of coefficients, $i = 0, \ldots, p_m$.

This model allows the international business cycle to respond to movements in global uncertainty, both through the conditional variances (contemporaneously, via movements in v_t) and through the conditional means (contemporaneously and with lags), via the coefficients collected in $\Pi_{m,i}$, $i = 0, \ldots, p_m$. In our implementation, we set the model's lag orders at $p = 2$ and $p_m = 2$.

2.2 Two-factor BVAR-GFSV model

As detailed in CCM2020, for the three-economy macroeconomic data set, our baseline results use a two-factor model with some restrictions. In particular, the model features two common factors in volatilities but includes only one of the factors in the conditional mean of the VAR and affecting the levels of the included variables. In addition, reflecting other evidence, the idiosyncratic component of volatility is simply a constant. The model takes the following form:

$$
\begin{aligned}
y_t &= \sum_{i=1}^{p} \Pi_i y_{t-i} + \sum_{i=0}^{p_m} \Pi_{m,i} \ln m_{t-i} + v_t && (6)\\
v_t &= A^{-1} \Lambda_t^{0.5} \epsilon_t, \ \ \epsilon_t \sim iid\ N(0, I) && (7)\\
\ln \lambda_{i,t} &= \beta_{m,i} \ln m_t + \beta_{f,i} \ln f_t + \ln h_i, \ \ i = 1, \ldots, n && (8)\\
\ln m_t &= \sum_{i=1}^{p_m} \delta_{m,i} \ln m_{t-i} + \delta'_{m,y} y_{t-1} + u_{m,t}, \ \ u_{m,t} \sim iid\ N(0, \phi_m) && (9)\\
\ln f_t &= \sum_{i=1}^{p_f} \delta_{f,i} \ln f_{t-i} + \delta'_{f,y} y_{t-1} + u_{f,t}, \ \ u_{f,t} \sim iid\ N(0, \phi_f). && (10)
\end{aligned}
$$

In this case, the log-volatility of each variable i follows a linear factor model with common unobservable uncertainty factors $\ln m_t$ and $\ln f_t$, which follow independent AR processes augmented to include y_{t-1}, and a constant idiosyncratic component $\ln h_i$. The volatility factors m_t and f_t are measures of (unobservable) global macroeconomic uncertainty. However, only the first global uncertainty measure, m_t, enters the conditional mean of the VAR and affects the levels of the macroeconomic variables contained in y_t, contemporaneously and with lags. The time-invariant idiosyncratic component captures differences in the average level of volatility across economies.

To spell out the notation, which follows that used in the one-factor model above, A is an $n \times n$ lower triangular matrix with ones on the main diagonal; Λ_t is a diagonal matrix of volatilities, $\lambda_{i,t}, i = 1, \ldots, n$; p denotes the number of y_t lags in the VAR; p_m denotes the number of $\ln m_t$ lags in the conditional mean of the VAR; Π_i is an $n \times n$ matrix, $i = 1, \ldots, p$; and $\Pi_{m,i}$ is an $n \times 1$ vector of coefficients, $i = 0, \ldots, p_m$. The uncertainty shocks $u_{m,t}$ and $u_{f,t}$ are independent of each other and independent of the conditional errors ϵ_t. For identification, we assume that $\ln m_t$ and $\ln f_t$ have zero unconditional means, fix their variances ϕ_m and ϕ_f at 0.03, and use a simple accept-reject step to restrict the first factor's loading on US GDP growth and the second factor's loading on EA GDP growth to be positive. In our implementation, we set the model's lag orders at $p = 2$, $p_m = 2$, and $p_f = 2$.

2.3 Estimation

Estimating the models with a Gibbs sampler requires the conditional posterior for the matrix of VAR coefficients Π (defined to collect all the coefficients in equation (6)). With smaller models, it is common to rely on a GLS solution for the posterior mean of the coefficient vector of the system of equations. However, such a system-of-equations approach slows considerably with larger models. In CCM2020, we instead estimated the VAR coefficients on an equation-by-equation basis, following a factorization of the posterior developed in CCM2019. Specifically, let $\pi^{(j)}$ denote the j-th column of the matrix Π, and let $\pi^{(1:j-1)}$ denote all of the previous columns. For each equation j, we drew $\pi^{(j)}$ from a multivariate Gaussian distribution with mean and variance as follows:

$$\bar{\mu}_{\pi^{(j)}} = \overline{\Omega}_{\pi^{(j)}} \left\{ \Sigma_{t=1}^{T} x_t \lambda_{j,t}^{-1} y_{j,t}^{*\prime} + \underline{\Omega}_{\pi^{(j)}}^{-1} (\underline{\mu}_{\pi^{(j)}}) \right\}, \tag{11}$$

$$\overline{\Omega}_{\pi^{(j)}}^{-1} = \underline{\Omega}_{\pi^{(j)}}^{-1} + \Sigma_{t=1}^{T} x_t \lambda_{j,t}^{-1} x_t', \tag{12}$$

where $y_{j,t}^{*} = y_{j,t} - (a_{j,1}^{*} \lambda_{1,t}^{0.5} \epsilon_{1,t} + \cdots + a_{j,,j-1}^{*} \lambda_{j-1,t}^{0.5} \epsilon_{j-1,t})$, with $a_{j,i}^{*}$ denoting the generic element of the matrix A^{-1} and $\underline{\Omega}_{\pi^{(j)}}^{-1}$ and $\underline{\mu}_{\pi^{(j)}}$ denoting the prior moments of the j-th equation, given by the j-th column of $\underline{\mu}_{\Pi}$ and the j-th block on the diagonal of $\underline{\Omega}_{\Pi}^{-1}$. Based on CCM2019, we intended for this approach to yield draws from the (correct) conditional posterior

$$\pi^{(j)} | \pi^{(1:j-1)}, A, \beta, f_{1:T}, m_{1:T}, h_{1:T}, y_{1:T} \sim \mathcal{N}(\bar{\mu}_{\pi^{(j)}}, \overline{\Omega}_{\pi^{(j)}}). \tag{13}$$

However, as follows from results in Bognanni (2021), drawing the VAR's coefficients in this way does not deliver the intended posterior distribution of the coefficient matrix. That is, drawing the coefficients as was done in CCM2018 does not actually sample from the density (13). As explained in more detail in Carriero, et al. (2021), the actual density associated with the original algorithm is missing a term, involving the information about $\pi^{(j)}$ contained in the most recent observations of the dependent variables of equations $j + 1, ..., n$.

To correctly use the information in question in an algorithm for sampling from the conditional posterior for the VAR's coefficients, Carriero, et al. (2021) propose using a sequence of Gibbs sampler draws. Specifically, in the model setting of CCM2020, one can correctly sample from the joint distribution $\Pi|A, \beta, f_{1:T}, m_{1:T}, h_{1:T}, y_{1:T}$ by cycling through the full conditional distributions

$$\pi^{(j)} \,|\, \pi^{(-j)}, A, \beta, f_{1:T}, m_{1:T}, h_{1:T}, y_{1:T} \tag{14}$$

for $j = 1, \ldots, n$, where $\pi^{(j)}$ is the j-th column of the $k \times n$ matrix Π — that is, the vector of coefficients appearing in equation j — and $\pi^{(-j)} = (\pi^{(1)\prime}, \ldots, \pi^{(j-1)\prime}, \pi^{(j+1)\prime}, \ldots, \pi^{(n)\prime})'$ collects all the coefficients in the remaining equations.

To establish this corrected approach, consider the triangular representation of the system:

$$\tilde{y}_t = Ay_t = A\Pi' x_t + \Lambda_t^{0.5}\epsilon_t = A(x_t'\Pi)' + \Lambda_t^{0.5}\epsilon_t, \tag{15}$$

which can be expressed as the following system of equations:

$$\begin{aligned}
\tilde{y}_{1,t} &= x_t'\pi^{(1)} + \lambda_{1,t}^{0.5}\epsilon_{1,t} \\
\tilde{y}_{2,t} &= a_{2,1}x_t'\pi^{(1)} + x_t'\pi^{(2)} + \lambda_{2,t}^{0.5}\epsilon_{2,t} \\
\tilde{y}_{3,t} &= a_{3,1}x_t'\pi^{(1)} + a_{3,2}x_t'\pi^{(2)} + x_t'\pi^{(3)} + \lambda_{3,t}^{0.5}\epsilon_{3,t} \\
&\vdots \\
\tilde{y}_{n,t} &= a_{n,1}x_t'\pi^{(1)} + \cdots + a_{n,n-1}x_t'\pi^{(n-1)} + x_t'\pi^{(n)} + \lambda_{n,t}^{0.5}\epsilon_{n,t},
\end{aligned} \tag{16}$$

with $\tilde{y}_t = Ay_t$ a vector with generic j-th element $\tilde{y}_{j,t} = y_{j,t} + a_{j,1}y_{1,t} + \cdots + a_{j,j-1}y_{j-1,t}$.

With this recursive system (16), it is evident that the coefficients $\pi^{(j)}$ of equation j influence not only equation j, but also the following equations $j+1, ..., n$, which is yet another way of seeing that these equations have some extra information about $\pi^{(j)}$ that the old algorithm missed. Importantly though, it remains true that the previous equations $1, ..., j-1$ have no information about the coefficients of equation j. With coefficient priors $\pi^{(j)} \sim N(\underline{\mu}_{\pi^{(j)}}, \underline{\Omega}_{\pi^{(j)}})$, $j = 1, ...n$, that are independent across equations (as is the case in all common VAR implementations), the first $j-1$ elements in the quadratic term above do not contain $\pi^{(j)}$. It follows that the conditional distribution $p(\pi^{(j)} \,|\, \pi^{(-j)}, A, \beta, f_{1:T}, m_{1:T}, h_{1:T}, y_{1:T})$ can be obtained using the subsystem composed of the last $n-j+1$ equations of (16).

In implementation, for drawing the coefficients of equation j, we use only equations j

and higher to sample $p(\pi^{(j)} \mid \pi^{(-j)}, A, \beta, f_{1:T}, m_{1:T}, h_{1:T}, y_{1:T})$:

$$\begin{aligned} z_{j,t} &= x_t' \pi^{(j)} + \lambda_{j,t}^{0.5} \epsilon_{j,t} \\ z_{j+1,t} &= a_{j+1,j}' x_t' \pi^{(j)} + \lambda_{j+1,t}^{0.5} \epsilon_{j+1,t} \\ &\vdots \\ z_{n,t} &= a_{n,j} x_t' \pi^{(j)} + \lambda_{n,t}^{0.5} \epsilon_{n,t}, \end{aligned}$$

where $z_{j+l,t} = \tilde{y}_{j+l,t} - \sum_{i \neq j, i=1}^{j+l} a_{j+l,i} x_t' \pi^{(i)}$, for $l = 0, ..., n-j$, and $a_{i,i} = 1$.

Then, using the above triangular representation, the full conditional distribution $(\pi^{(j)} \mid \pi^{(-j)}, A, \beta, f_{1:T}, m_{1:T}, h_{1:T}, y_{1:T})$ is

$$(\pi^{(j)} \mid \pi^{(-j)}, A, \beta, f_{1:T}, m_{1:T}, h_{1:T}, y_{1:T}) \sim \mathcal{N}(\overline{\mu}_{\pi^{(j)}}, \overline{\Omega}_{\pi^{(j)}}),$$

where

$$\overline{\Omega}_{\pi^{(j)}}^{-1} = \underline{\Omega}_{\pi^{(j)}}^{-1} + \sum_{i=j}^{n} a_{i,j}^2 \sum_{t=1}^{T} \frac{1}{\lambda_{i,t}} x_t x_t', \tag{17}$$

$$\overline{\mu}_{\pi^{(j)}} = \overline{\Omega}_{\pi^{(j)}} \left(\underline{\Omega}_{\pi^{(j)}}^{-1} \underline{\mu}_{\pi^{(j)}} + \sum_{i=j}^{n} a_{i,j} \sum_{t=1}^{T} \frac{1}{\lambda_{i,t}} x_t z_{i,t} \right). \tag{18}$$

As documented in Carriero, et al. (2021), this approach preserves the gains in computational complexity described in CCM2019. Although the use of additional information (data) for all but the n-th equation makes this algorithm empirically slower than that originally used in the paper, in application the computational time is comparable. Accordingly, in this note, we use this approach to sampling the VAR's coefficients to correct and update the results of CCM2020.[1]

3 Corrected results

In general, the correction of the estimation algorithm has proven to make it somewhat more challenging to use GFSV specifications to estimate measures of international uncertainty and their effects. Some estimation challenges are to be expected, given the comovement of forecast error variances across the variables of the models, the counter-cyclicality of uncertainty, the non-linear features of the models, and the large size of the models. The algorithm

[1] See Carriero, et al. (2021) for an implementation of computations that makes use of a data-matrix type of notation that is easy to implement and computationally efficient in programming languages such as Matlab.

correction seems to have made these challenges steeper, for reasons not easy to pinpoint. For example, with some of the loose prior settings of CCM2020, estimates with the new algorithm showed more issues with mixing and convergence of the MCMC chain.

Accordingly, to be able to reliably estimate the model with the corrected algorithm, we have made a few prior changes relative to the settings of CCM2020. For the simpler case of the 19-country model with a single uncertainty factor, we made one change, to make the prior on the rows a_j of the matrix A more informative (but not tight), lowering the prior variance on each element from 10 to 0.5. For the very large model of the three-economy application, we made the prior on the rows a_j of the matrix A still more informative, lowering the prior variance on each element from the paper's old setting of 10 to a new setting of 0.05 (the model estimates indicate that this is not so tight as to simply make the posterior the same as the prior). For this model, we also lowered the hyperparameter θ_3 governing shrinkage of the factor coefficients in the VAR's conditional mean from the paper's setting of 10 to a more modestly informative setting of 1. In addition, for the loading $\beta_{f,i}$, $i = 1, \ldots, n$, on the second uncertainty factor $\ln f_t$, we slightly tightened the prior standard deviation, lowering it from the paper's 1.0 to 0.5. For the coefficients on y_{t-1} in the processes of the factors, we tightened the prior standard deviation from the paper's 0.4 to 0.2. Finally, for the idiosyncratic volatility component, we tightened the prior variance, lowering it from the paper's 2.0 to 1.0.

In the remainder of this note, we provide results corresponding to those in CCM2020, but using the corrected algorithm for the VAR estimation described above. In general, the corrected results are qualitatively the same as those provided in CCM2020.

3.1 Commonality in international uncertainty

To assess the global factor structure of macroeconomic uncertainty, we apply to estimates of the stochastic volatilities of BVARs some basic factor model diagnostics. The volatility estimates are posterior medians of log stochastic volatilities obtained from conventional BVARs with stochastic volatility. In corrected estimates for GDP growth in 19 countries, the measures of factor structure continue to suggest one strong factor in the international volatility of the business cycle. The first factor accounts for an average of about 75 percent of the variation in log volatilities. The second and third factors account for about 13 and 7 percent, respectively. The Ahn-Horenstein ratio peaks at one factor. As reported in Table 1, the factor loadings associated with the principal components are fairly tightly clustered around 1. In this sense the common volatility factor puts comparable weight on each country's volatility (with the exception of Norway).

For the larger set of macroeconomic indicators for the US, EA, and UK, we use volatility estimates from BVAR-SV models fit separately for each economy to assess the commonality in volatility. Figure 1 compares volatility estimates across these three economies for a subset of major macroeconomic indicators. In this comparison, volatility is reported in the way common in the literature, as the (posterior median of the) standard deviation of the reduced-form innovation in the BVAR. Qualitatively, these estimates are similar to those of the paper, pointing to considerable commonality within and across countries. As the chart indicates, for a given country, there is significant comovement across variables. For example, for the US, most variables display a rise in volatility around the recessions of the early 1990s, 2001, and 2007-2009. For the EA, most variables display sizable increases in uncertainty in the early and mid-1990s and again with the Great Recession. In addition, there appears to be significant comovement across economies, somewhat more so for volatility in the US and EA than in the case of the UK.

In updated estimates for the three-economy macroeconomic data set, a first factor accounts for an average of 39 percent of the variation in log volatilities, comparable to the result indicated in the paper. For most variables, the estimated loadings on this factor reported in Table 2 are clustered around a value of 1. For example, the loadings on GDP growth are 1.394 for the US, 1.240 for the EA, and 1.283 for the UK. In this sense the common volatility factor puts comparable (but not equal) weight on the volatility of most variables in the model. A second factor accounts for about 24 percent of the variation in international macroeconomic volatility. Together, two factors account for about 63 percent of the variation in volatility across indicators and countries. Subsequent factors account for significantly smaller marginal shares of variation. The Ahn-Horenstein ratio peaks at two factors. Together, the R^2 and Ahn-Horenstein estimates suggest two factors in this larger data set.

3.2 BVAR-GFSV estimates of uncertainty

Although the BVAR-GFSV estimates of uncertainty reflect influence from the first moments of macroeconomic data, the estimates are also directly related to the loadings on the common factor in volatility. These loadings (for the three-economy macroeconomic data set, we report only the first factor's loadings for brevity) are reported in the last columns of Tables 1 and 2. In the case of the 19-country GDP data set, the loadings are broadly centered around 1, with a minimum of 0.353 for Switzerland and a maximum of 1.640 for Germany. In this respect, the loadings estimated from the BVAR-GFSV model are similar to those estimated by principal components applied to log volatilities of the BVAR-SV model. In the case of the three-economy macroeconomic data set, most of the variables have sizable loadings on the volatility factor (keeping in mind that the scale of the loadings reflects the normalization imposed by fixing the innovation variance for identification). Across variables, the average of the loading estimates (posterior means) is 0.80, with a range of 0.17 to 1.51; more than 3/4 of the loadings are above 0.5.

Figure 2 displays the posterior distribution of the measures of uncertainty obtained from the BVAR-GFSV specification, along with corresponding measures obtained from the first principal component of the log volatilities from the BVAR-SV models. The top panel provides estimates for the 19-country GDP data set, and the bottom panel reports estimates for the three-economy macroeconomic data set. In reporting the BVAR-GFSV estimates, we define uncertainty as the square root of the common volatility factor ($\sqrt{m_t}$), corresponding to a standard deviation. Figure 2 also reports the 15 percent-85 percent credible set bands around our estimated measure of uncertainty, which is correctly considered a random variable in our approach. In the case of the first principal component of BVAR-SV log volatilities, for scale comparability we exponentiate the principal component and then compute (and plot) its square root.

Although not covered in charts in the interest of brevity, the corrected estimates of global uncertainty are very similar to the original results in the paper. In the GDP-only data set for 19 countries, the correlation of the corrected factor with the original is 0.97. In the three-economy macroeconomic data set, the corresponding correlation for the first factor is 0.96, and the correlation between the corrected and original estimates of the second factor is 0.75.

Moreover, as indicated in Figure 2, in corrected results similar to the original results in the paper, the uncertainty factors show significant increases around some of the political and economic events that Bloom (2009) highlights as periods of uncertainty, including the first Gulf war, 9/11, the Enron scandal, the second Gulf war, and the recent financial crisis period. In some cases, increases in uncertainty around such events seem to be defined somewhat more clearly in our larger variable set (bottom panel) than in the GDP-only data set for 19 countries. But in both cases, the credible sets around the BVAR-GFSV estimates indicate that the uncertainty around uncertainty estimates is sizable. Although we believe it to be important to take account of such uncertainty around uncertainty measures, the estimates obtained with our BVAR-GFSV model are significantly correlated with those obtained from the principal component of the BVAR-SV volatility estimates, more so in the three-economy macroeconomic data set (correlation of 0.703) than in the 19-country GDP data set (correlation of 0.546).

Figure 3 compares our uncertainty estimates to each other and to other estimates in the literature, including CCM macro and financial uncertainty from Carriero, Clark, and Marcellino (2018; also corrected for the algorithm issue noted above); JLN macro and financial uncertainty from Jurado, Ludvigson, and Ng (2015); global economic policy uncertainty (EPU) from Davis (2016); common uncertainty from Mumtaz and Theodoridis (2017); and common uncertainty from Berger, Grabert, and Kempa (2016). As indicated in the top left panel, even though our three-economy macroeconomic and 19-economy GDP data sets differ significantly in composition, estimates of uncertainty obtained with our BVAR-GFSV model are quite similar, with a correlation of 0.730. The estimate from our three-economy data set is also significantly correlated with the estimate of US macroeconomic uncertainty from Carriero, Clark, and Marcellino (2018) and to a slightly lesser extent with the Jurado, Ludvigson, and Ng (2015) estimate of US macroeconomic uncertainty. This suggests that global macroeconomic uncertainty is closely related to uncertainty in the US, which might not seem surprising given the tie of the international economy to the US economy. On the other hand, we have noted that most variables have significant loadings on the international uncertainty factors. So by this very simple measure, the uncertainty we capture is global and not specific to the US.

Our estimate of global macroeconomic uncertainty appears to be modestly correlated with estimates of financial uncertainty from the literature and the global economic policy uncertainty measure of Davis (2016). Our estimate of global macroeconomic uncertainty is also only modestly correlated with the uncertainty measures of Berger, Grabert, and Kempa (2016) and Mumtaz and Theodoridis (2017), both of which display relatively sharp spikes with the Great Recession. Although the number of differences across specifications makes it difficult to identify which factor might account for the differences in uncertainty estimates, one probably important difference is that our uncertainty measure is a common factor in macroeconomic volatilities, whereas in these papers uncertainty is the volatility of common factors in the business cycle.

3.3 Measuring the impact of uncertainty: Impulse response estimates and historical decompositions from BVAR-GFSV model

Figures 4 and 5 provide updated BVAR-GFSV estimates of impulse response functions for a shock to international macroeconomic uncertainty, which yield results similar to those in the paper. Starting with the 19-country results in Figure 4, an international shock to macroeconomic uncertainty slowly dies out over several quarters. The rise in uncertainty induces statistically significant, persistent declines in GDP in most of the countries. For example, after several quarters, GDP falls about 0.5 percentage point in countries including the US, Canada, France, the Netherlands, and the UK.

For space saving and readability, Figure 5 covers a subset of variables in providing impulse response estimates for the three-economy macroeconomic data set, and it reports posterior medians and 70 percent credible sets for the US responses but just posterior medians for the EA and UK. In the estimates for this data set, it is also the case that an international shock to macroeconomic uncertainty (to the factor $\ln m_t$ in the VAR's conditional mean) gradually dies out over a few quarters. For the US, EA, and UK, the heightened international uncertainty reduces GDP and components including investment, exports, and imports. In all three economies, employment falls and unemployment rises, and some other measures of economic activity, including confidence or sentiment indicators and capacity utilization, also fall. The shock does not have any consistently significant and negative effects on producer or consumer prices; compared to the original results, the revised estimates show more increases rather than decreases in price levels. Although stock prices fall in all three economies, the policy rate falls in the US but rises in the EA and is little changed in the UK.

Although these impulse responses show that shocks to uncertainty have significant effects, they cannot provide an assessment of the broader cyclical importance of global macroeconomic uncertainty shocks. For that broader assessment, we estimate historical decompositions. Figures 6 (19-country GDP data set) and 7 (three-economy macroeconomic data set) show the standardized data series, the direct contributions of shocks to macroeconomic uncertainty, and the direct contributions of the VAR's shocks. The reported estimates are posterior medians of decompositions computed for each draw from the posterior. To save space, the charts provide results for a subset of selected variables. Finally, the decomposition results start in 1987:Q1 for the 19-country GDP data set and, for better readability, 1998:Q1 for the three-economy macroeconomic data set.

As indicated in Figure 6's decomposition estimates for the 19-country GDP data set, while shocks to uncertainty can have noticeable effects on GDP growth in many countries, on balance they are not a primary driver of fluctuations in growth. For example, over the period of the Great Recession and subsequent recovery, shocks to uncertainty made modest contributions to the paths of GDP growth in many countries (e.g., US, France, Spain, and Sweden) and small contributions in some countries (e.g., Japan and Norway). In the declines of GDP growth observed in a number of countries in the early 1990s and early 2000s, uncertainty shocks made small contributions in some countries (e.g., US, Sweden, and UK). Overall, shocks to the VAR's variables played a much larger role than did uncertainty shocks.

Figure 7's decomposition estimates for the three-economy macroeconomic data set paint a broadly similar picture. For example, around the Great Recession (2007-2009 for the US), shocks to macroeconomic uncertainty (the first factor $\ln m_t$) contribute a little to the variation in fluctuations in economic activity, including in GDP and housing investment, but not much to fluctuations in a number of other variables, including inflation and stock prices. Similar patterns are evident in the decline in GDP growth observed in the early 2000s. With this data set, too, the effects of uncertainty shocks are generally dominated by the contributions of the VAR's shocks.

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Table 1: Factor loadings: 19-country GDP data set

Country	Principal component loading	GFSV loading posterior mean (st. dev.).
US	1.054	0.894 (0.365)
Australia	1.109	0.614 (0.361)
Austria	1.118	1.107 (0.393)
Belgium	1.007	1.272 (0.398)
Canada	1.134	0.988 (0.424)
Denmark	0.674	0.464 (0.415)
Finland	1.074	1.107 (0.364)
France	1.048	0.797 (0.420)
Germany	1.135	1.640 (0.372)
Italy	1.133	1.056 (0.378)
Japan	1.106	0.672 (0.396)
Luxembourg	0.773	1.084 (0.352)
Netherlands	0.870	0.863 (0.403)
Norway	-0.025	0.493 (0.413)
Portugal	1.006	1.228 (0.415)
Spain	0.919	1.426 (0.410)
Sweden	1.141	1.072 (0.394)
Switzerland	0.911	0.353 (0.407)
UK	1.125	1.056 (0.425)

Note: The second column provides loadings on a first common factor estimated as the principal component of log volatilities of a BVAR-SV model. The third column provides estimates of the loadings $\beta_{m,i}$ of equation (2) of the one-factor BVAR-GFSV model.

Table 2: Loadings on first factor: three-economy macroeconomic data set

Country	Principal component loading	GFSV loading posterior mean (st. dev.)	Country	Principal component loading	GFSV loading posterior mean (st. dev.)
US real GDP	1.394	0.734 (0.296)	EA employment	0.784	0.850 (0.457)
US real consumption	1.153	0.702 (0.325)	EA unemployment rate	1.028	1.122 (0.390)
US real government consumption	1.422	0.566 (0.311)	EA Eonia rate	0.748	1.327 (0.394)
US real investment	1.322	0.445 (0.351)	EA 2-year bond yield	0.526	0.845 (0.381)
US real exports	1.360	0.659 (0.331)	EA 10-year bond yield	0.970	0.614 (0.393)
US real imports	1.454	0.582 (0.341)	EA M3	1.124	0.776 (0.383)
US real inventories	1.213	0.704 (0.406)	EA GDP deflator	0.613	0.689 (0.376)
US unit labor costs	0.834	0.926 (0.333)	EA consumer prices	1.102	0.848 (0.369)
US employment	1.235	0.187 (0.387)	EA core consumer prices	-0.103	0.644 (0.389)
US hours worked	-1.100	0.174 (0.300)	EA producer prices	1.075	0.791 (0.421)
US unemployment rate	1.398	0.857 (0.382)	EA real housing investment	1.070	1.418 (0.370)
US federal funds rate	1.398	1.239 (0.356)	EA stock price index	0.430	0.529 (0.411)
US 2-year bond yield	0.598	0.860 (0.381)	EA capacity utilization	0.551	0.782 (0.406)
US 10-year bond yield	0.585	0.650 (0.383)	EA consumer confidence	-1.120	0.641 (0.413)
US M2	1.011	0.751 (0.399)	EA industrial confidence	0.890	0.301 (0.422)
oil price	1.387	1.199 (0.312)	EA purchasing managers' index	-0.939	0.270 (0.410)
commodity prices	1.367	1.277 (0.286)	EA labor shortages	-0.074	1.508 (0.400)
US consumer prices	1.226	1.148 (0.308)	UK real GDP	1.283	0.979 (0.390)
US core consumer prices	1.294	0.935 (0.340)	UK real consumption	0.707	0.831 (0.369)
US producer prices	-0.696	0.681 (0.375)	UK real government consumption	0.261	0.545 (0.360)
US real housing investment	0.881	0.801 (0.397)	UK real investment	1.163	0.398 (0.351)
US stock price index	1.088	0.884 (0.353)	UK real exports	-0.053	0.667 (0.388)
US capacity utilization	1.401	0.794 (0.356)	UK real imports	0.333	0.665 (0.360)
US consumer confidence	1.221	1.106 (0.375)	UK unit labor costs	0.344	0.340 (0.375)
US industrial confidence	1.315	0.675 (0.370)	UK industrial confidence	-1.254	0.538 (0.396)
US purchasing managers' index	1.470	1.152 (0.372)	UK consumer confidence	0.783	1.015 (0.394)
EA real GDP	1.240	1.487 (0.338)	UK employment	0.589	0.845 (0.365)
EA real consumption	0.863	0.895 (0.375)	UK unemployment rate	-0.603	0.749 (0.399)
EA real government consumption	0.493	0.397 (0.416)	UK producer prices	1.246	1.034 (0.386)
EA real investment	0.698	0.207 (0.393)	UK retail price index	1.170	1.217 (0.403)
EA real exports	0.913	0.921 (0.376)	UK official bank rate	0.833	0.948 (0.408)
EA real imports	0.977	0.829 (0.382)	UK 10-year bond yield	-0.010	0.861 (0.409)
EA real inventories	0.761	0.747 (0.389)	UK stock price index	0.474	0.935 (0.411)
EA unit labor costs	0.869	0.784 (0.369)			

Note: The second and fifth columns provide loadings on a first common factor estimated as the principal component of log volatilities of BVAR-SV models estimated for each economy. The third and sixth columns provide estimates of the loadings $\beta_{m,i}$ of equation (9) of the two-factor BVAR-GFSV model.

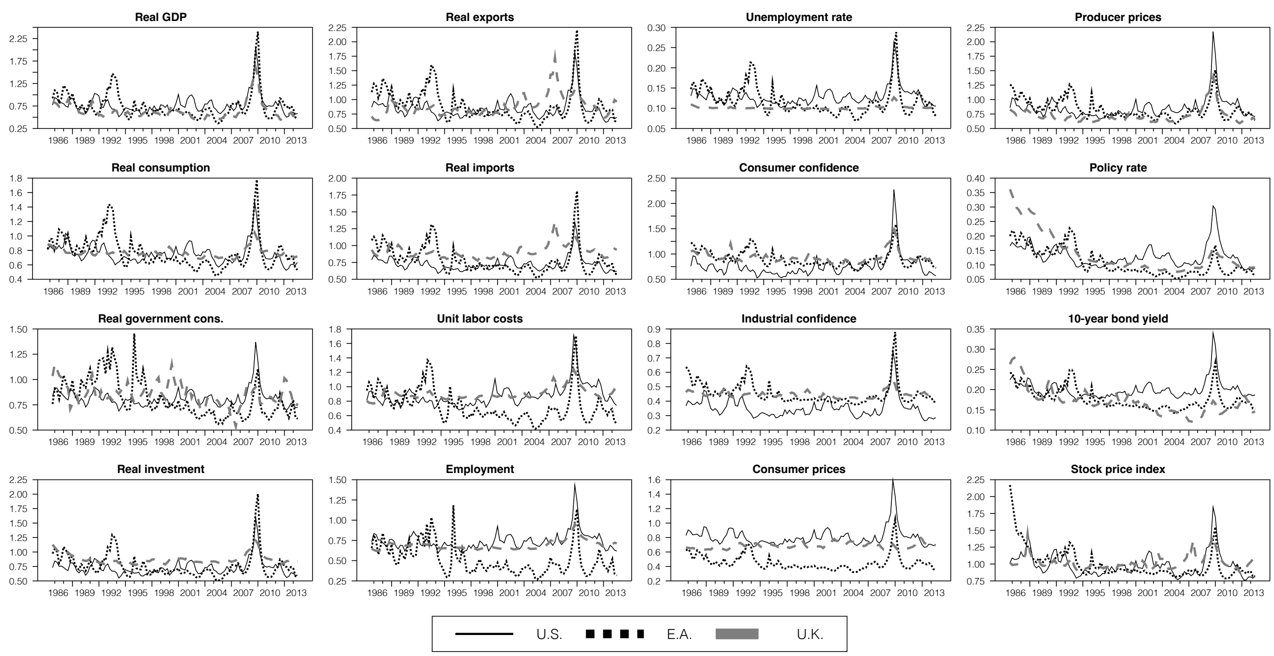

Figure 1: BVAR-SV estimates of volatilities, selected variables. The reported entries are posterior medians of standard deviations of reduced-form innovations from BVAR-SV models estimated for each economy.

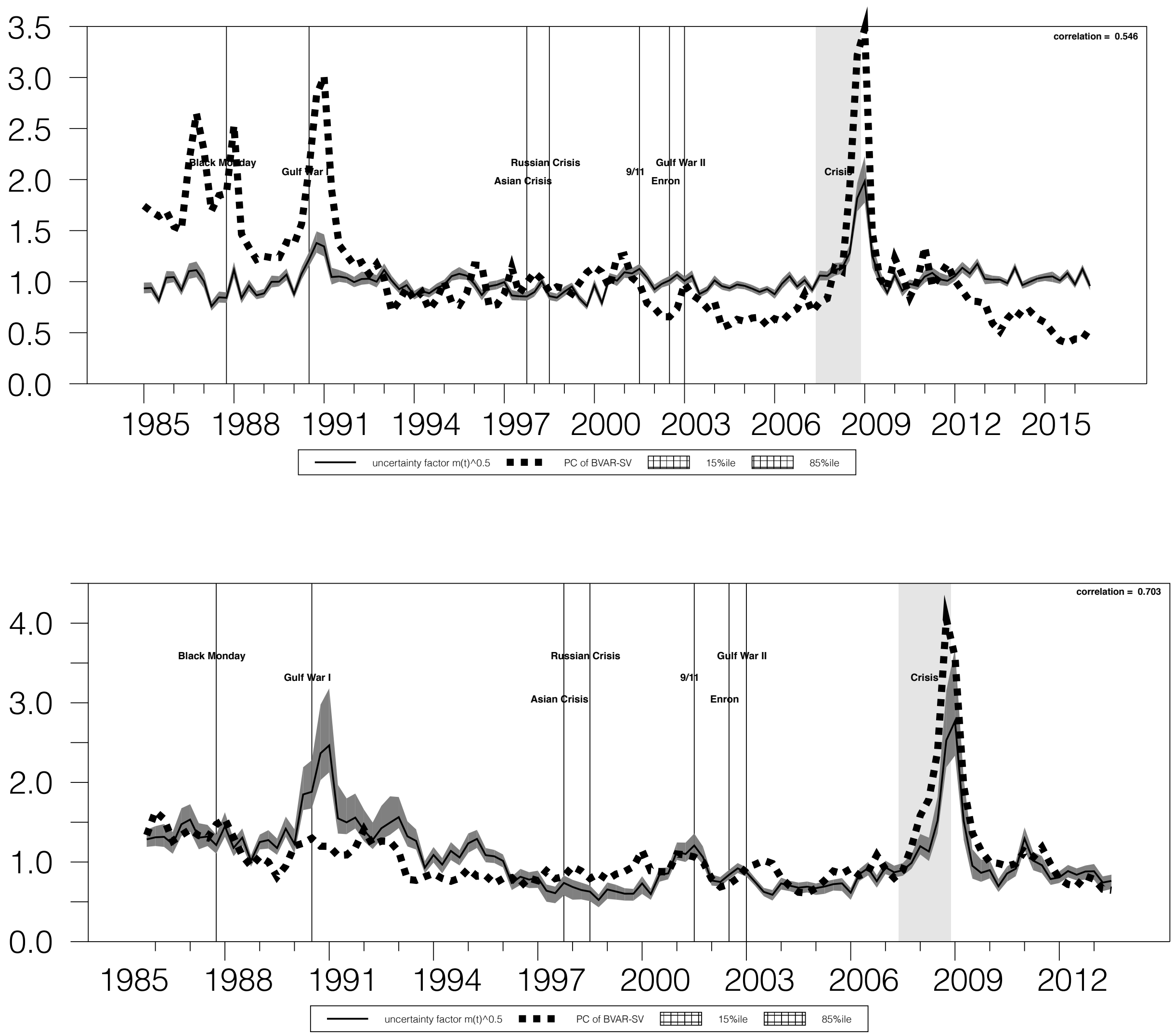

Figure 2: Uncertainty estimates for 19-country GDP data set in the top panel and for three-economy macroeconomic data set in the bottom panel. In each panel, the dotted line provides an estimate obtained from the first principal component of the BVAR-SV estimates of log volatility. The solid black line and gray-shaded regions provide the posterior median and 15%/85% quantiles of the BVAR-GFSV estimate of macroeconomic uncertainty ($m_t^{0.5}$). The periods indicated by black vertical lines or regions correspond to the uncertainty events highlighted in Bloom (2009). Labels for these events are indicated in text horizontally centered on the event's start date.

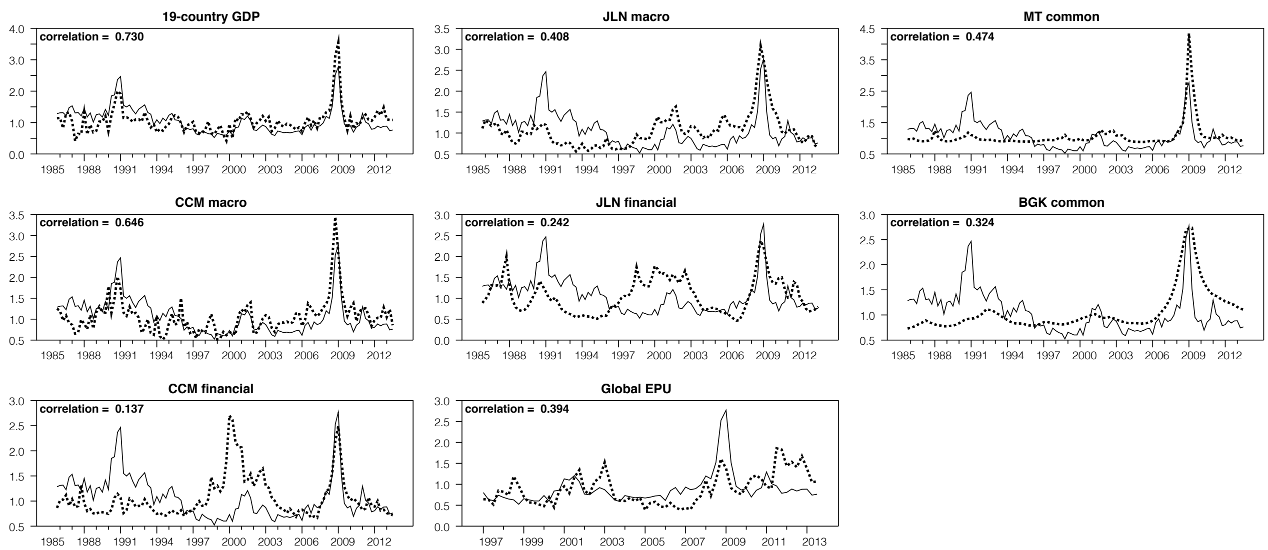

Figure 3: Comparison of uncertainty estimates to others in the literature. The top left panel compares the uncertainty estimate obtained from the three-economy macroeconomic data set (solid line) to that obtained with the 19-country GDP data set (dotted line). Other panels compare the three-economy macroeconomic data set estimate (solid line) to a different estimate (dotted line) from the literature, normalized to have the same mean and variance as the three-economy macroeconomic data set estimate.

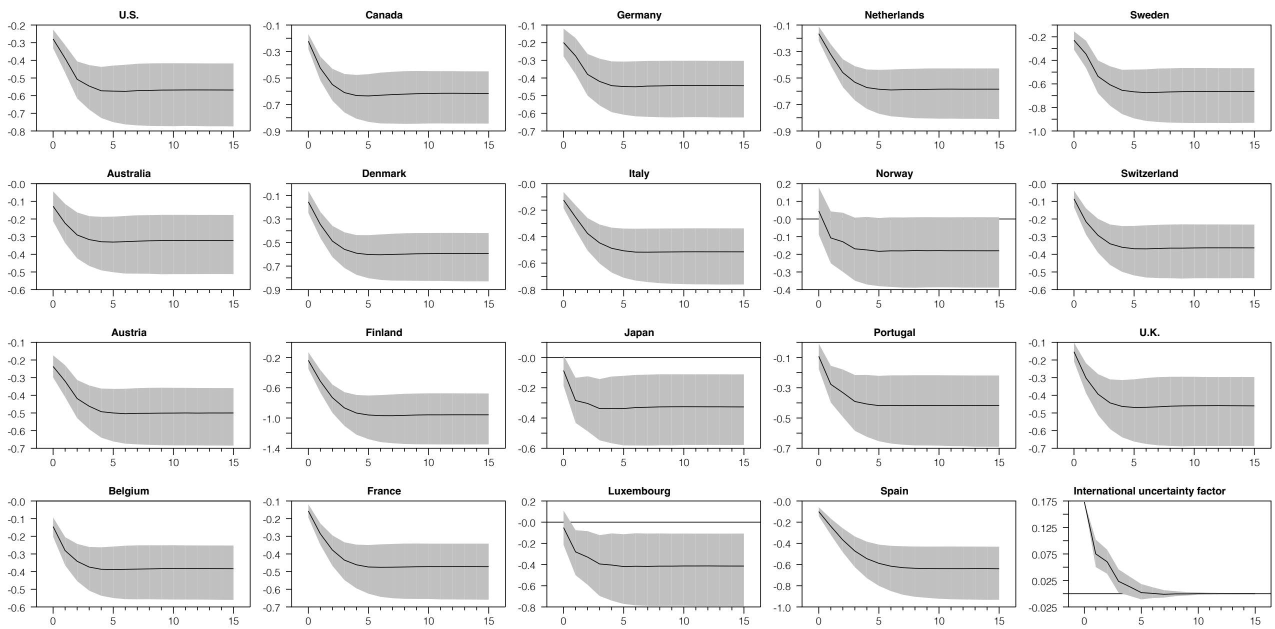

Figure 4: Impulse responses for international uncertainty shock: one-factor BVAR-GFSV estimates for 19-country GDP data set, posterior median (black line) and 15%/85% quantiles

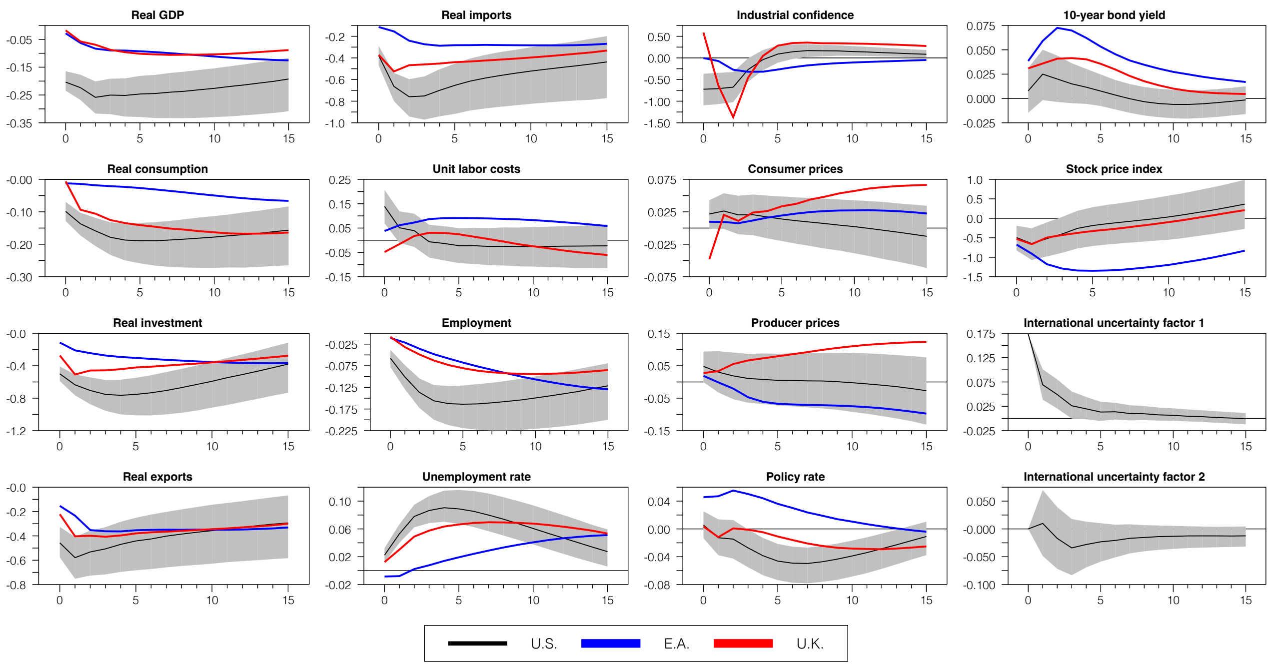

Figure 5: Impulse responses for international uncertainty shock: two-factor BVAR-GFSV estimates for three-economy macroeconomic data set, selected variables. The black line and gray shading provide posterior medians and 15%/85% quantiles for the US response. The dotted lines provide posterior medians for the EA and UK.

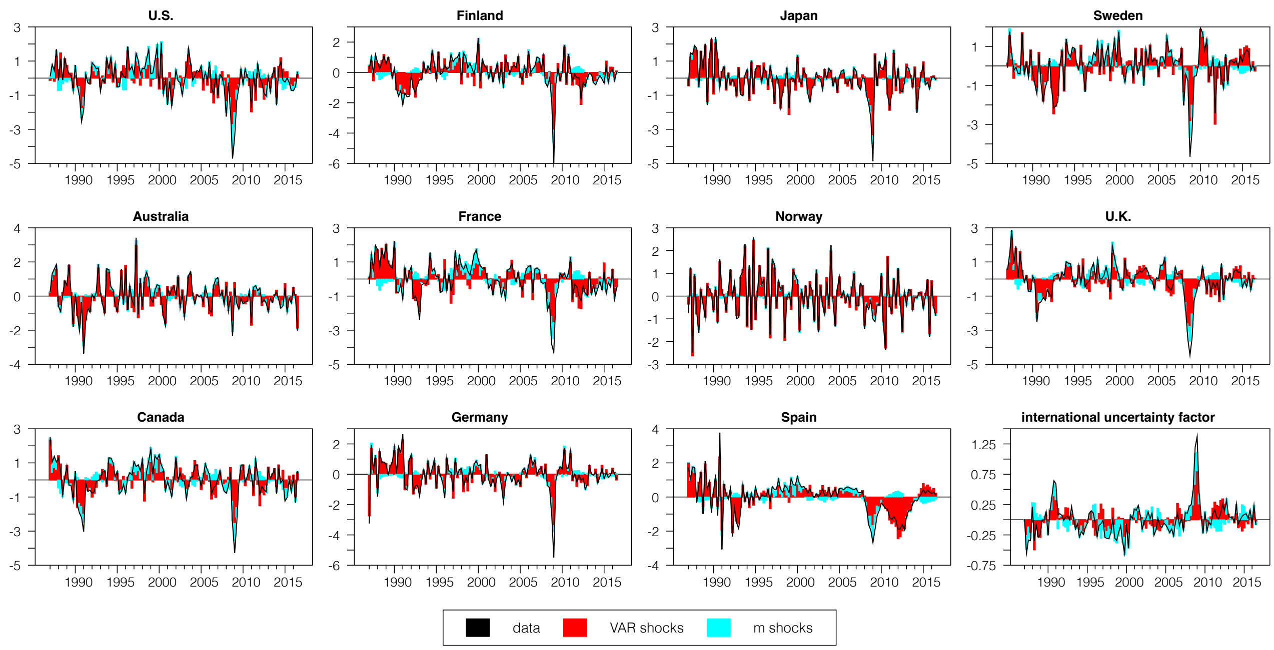

Figure 6: Historical decompositions: one-factor BVAR-GFSV estimates for 19-country GDP data set, selected variables, posterior medians

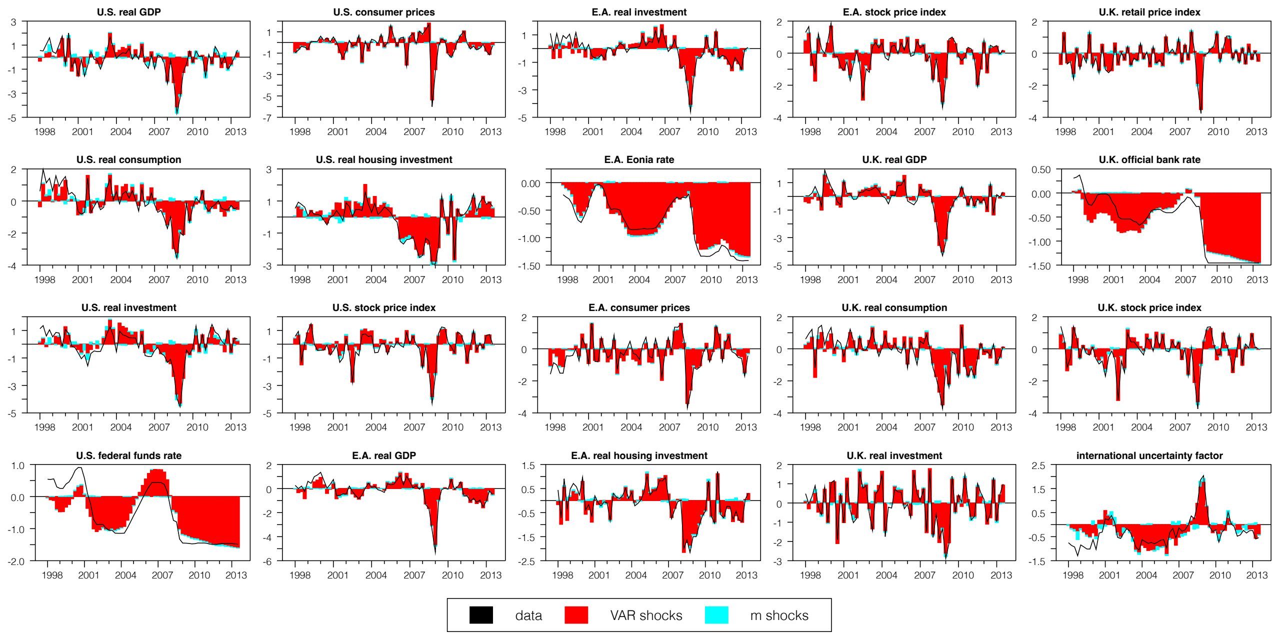

Figure 7: Historical decompositions: two-factor BVAR-GFSV estimates for three-economy macroeconomic data set, selected variables, posterior medians