Appendix to Structural FECM: Cointegration in large-scale structural FAVAR models

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A Impulse response analysis in the FECM and FAVAR - an analytical illustration

We illustrate analytically the computation of structural responses using the FECM rather than the FAVAR with a simple but comprehensive example. The example may easily be seen to be a special case of the general specification introduced in the main text, obtained by restricting the dimension of the factor space and of the variables of interest studied.

We suppose that the large information set available can be summarized by one $I(1)$ common factor, f, and that the econometrician is particularly interested in the response of one of the many variables, x_1, and that she can choose any of the three following models. First, a FECM, where the explanatory variables of the FAVAR are augmented with a term representing the (lagged) deviation from the long run equilibrium of x_1 and f. Second, a FAVAR model where the change in x_1 (Δx_1) is explained by an infinite number of its own lags and by lags of the change in f. And, third, the same model but with a finite number of lags. We want to compare the differences in IRFs resulting from the three models.

To start with, let us consider a system consisting of the two variables x_1 and x_2 and of one factor f. The factor follows a random walk process,

$$f_t = f_{t-1} + \varepsilon_t, \tag{1}$$

where ε_t is a structural shock and we are interested in the dynamic response to this shock. The factor loads directly on x_2,

$$x_{2t} = f_t + u_t, \tag{2}$$

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while the process for x_1 is given in ECM form as

$$\Delta x_{1t} = \alpha\left(x_{1t-1} - \beta f_{t-1}\right) + \gamma \Delta f_{t-1} + v_t, \quad \alpha < 0. \tag{3}$$

or

$$\Delta x_{1t} = \alpha\left(x_{1t-1} - \beta f_{t-1}\right) + \gamma \varepsilon_{t-1} + v_t. \quad \alpha < 0 \tag{4}$$

Here the processes ε_t and v_t are assumed $i.i.d.(0, I_N)$, while u_t is allowed to have a moving average structure, i.e. $u_t = u_t^* / \left(1 - \eta L\right)$, $|\eta| < 1$ and u_t^* is $i.i.d.(0, \sigma^2_{u^*})$ Hence, the DGP is a FECM.

Note that the moving-average representation of x_{1t} can be written as

$$\begin{aligned}
x_{1t} &= (1+\alpha)^h x_{1t-h} \\
&\quad + (1+\alpha)^{h-1}(-\alpha\beta(\varepsilon_{t-h} + \varepsilon_{t-h-1} + ... + \varepsilon_{-h}) + +\gamma\varepsilon_{t-h} + v_{t-h+1}) \\
&\quad + (1+\alpha)^{h-2}(-\alpha\beta(\varepsilon_{t-h+1} + \varepsilon_{t-h} + ... + \varepsilon_{-h+1}) + \gamma\varepsilon_{t-h+1} + v_{t-h+2}) \\
&\quad \vdots \\
&\quad - (\alpha\beta(\varepsilon_{t-1} + \varepsilon_{t-2} + ... + \varepsilon_1) + \gamma\varepsilon_{t-1} + v_t.
\end{aligned}$$

Based on this, the impulse response function takes the following form:

$$\frac{\partial \Delta x_{1t+h}}{\partial \varepsilon_t} = \frac{\partial x_{1t+h}}{\partial \varepsilon_t} - \frac{\partial x_{1t+h-1}}{\partial \varepsilon_t} = -(1+\alpha)^{h-1}\alpha\beta + \alpha(1+\alpha)^{h-2}\gamma.$$

The FECM representation of x_1 can also be written as a FAVAR. In fact, since the error-correction term $x_{1t} - \beta f_t$ evolves as

$$\begin{aligned}
x_{1t} - \beta f_t &= (\alpha+1)\left(x_{1t-1} - \beta f_{t-1}\right) + \gamma\Delta f_{t-1} + v_t - \beta\varepsilon_t \\
&= \frac{\gamma \Delta f_{t-1}}{1 - (\alpha+1) L} + \frac{v_t - \beta\varepsilon_t}{1 - (\alpha+1) L},
\end{aligned}$$

we can re-write equation (3) as

$$\Delta x_{1t} = \gamma \Delta f_{t-1} + \frac{\alpha\gamma\Delta f_{t-2}}{1 - (\alpha+1) L} + v_t + \frac{\alpha\left(v_{t-1} - \beta\varepsilon_{t-1}\right)}{1 - (\alpha+1) L}, \tag{5}$$

which is a FAVAR of infinite order. The corresponding moving-average representation then follows directly as

$$\Delta x_{1t} = \gamma \varepsilon_{t-1} + \frac{\alpha\gamma\varepsilon_{t-2}}{1 - (\alpha+1) L} + v_t + \frac{\alpha\left(v_{t-1} - \beta\varepsilon_{t-1}\right)}{1 - (\alpha+1) L}. \tag{6}$$

This implies that the impulse responses of the infinite-order FAVAR model would be

$$\frac{\partial \Delta x_{1t+h}}{\partial \varepsilon_t} = -(1+\alpha)^{h-1}\alpha\beta + \alpha(1+\alpha)^{h-2}\gamma.$$

We therefore see that only using a FAVAR with an infinite number of lags allows us to recover the same IRFs as in the FECM. However, in practice, a short lag length is used in the FAVAR, so that the resulting responses will be different from those from the FECM, the more so the poorer the finite lag approximation is to the infinite order FAVAR.

A simulation experiment whose design is based on a frequently-used panel of US macroeconomic data, presented in Table 1 in Section 5, reveals that the differences in the impulse responses obtained by the FECM and the (finite order) FAVAR can be substantial.

B Effects of permanent productivity shocks: DSGE evidence and robustness checks

Figure B.1 presents the robustness check of impulse responses to a permanent producitivty shock identified through long-run restrictions with respect to the number of estimated factors. In particular, we vary the number of static factors q from 4 to 7. The Stock and Watson (2005) test for these cases signals also an equal number of dynamic factors. To indicate estimation uncertainty, the shaded areas are the bootstrapped confidence intervals (computed in the same way as above i.e. by resampling both the innovations to factors and the idiosyncratic errors) of the impulse responses of the model with fice factors ($q = 5$). In addition Figure B.2 reports the responses to a positive and permanent productivity shock obtained by an estimated DSGE model of the Euro area of Adolfson et al. (2007). Broad coherence between the impulse responses obtained with the FECM and the DSGE reinforces the intepretation of empirically identified real shocks as permanent productivity shocks.

Figure B.1: Impulse responses to real stochastic trend in the US conditional on the number of estimated factors

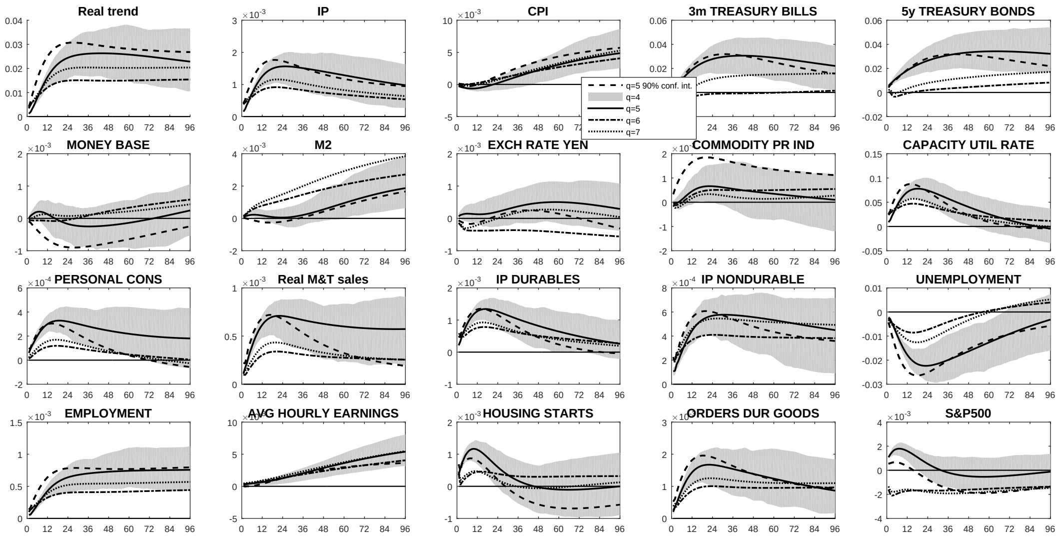
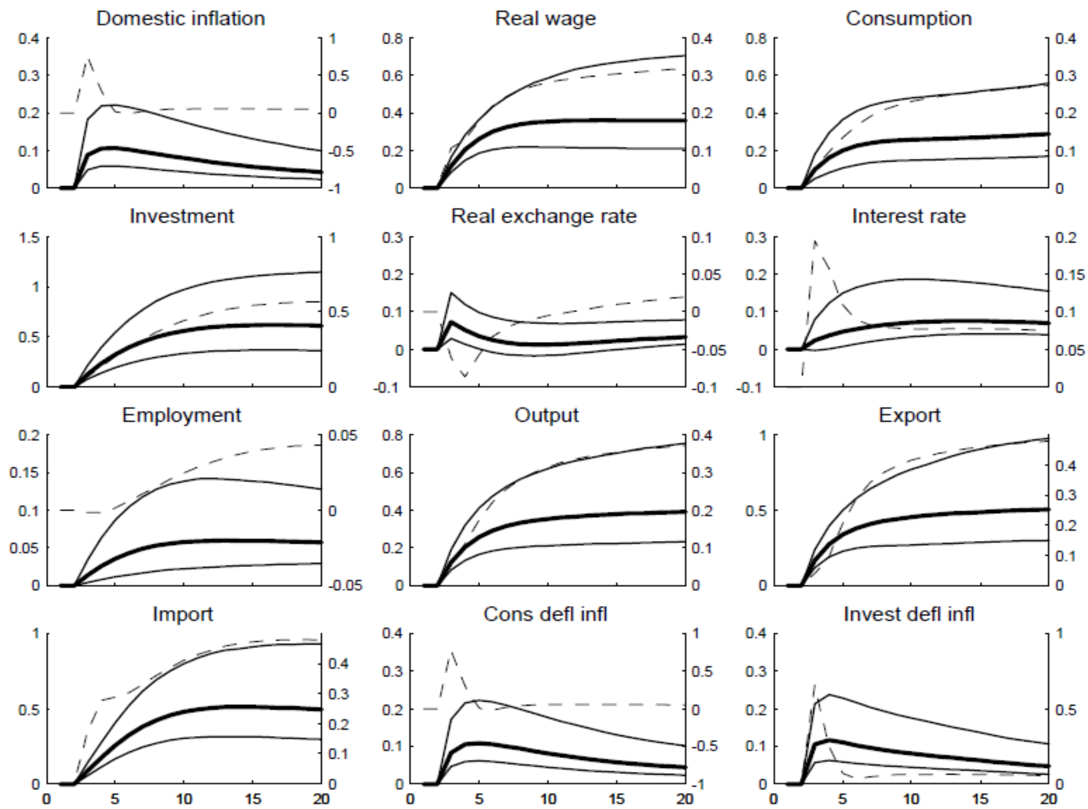


Figure B.2: Impulse responses to a permanent technology shock for the EA from Adolfson et al. (2007)



Note: Benchmark impulse responses under price rigidity and imperfect exchange rate (pass-through solid, left axis) and flexible prices and wages (dashed, right axis).