

Online Appendix for:  
**ON THE LOW-FREQUENCY RELATIONSHIP  
BETWEEN PUBLIC DEFICITS AND INFLATION**

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May 22, 2014

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# 1 Estimation

## 1.1 Data

In the following we plot the main raw time series which we use for the estimation calculations presented in the paper.

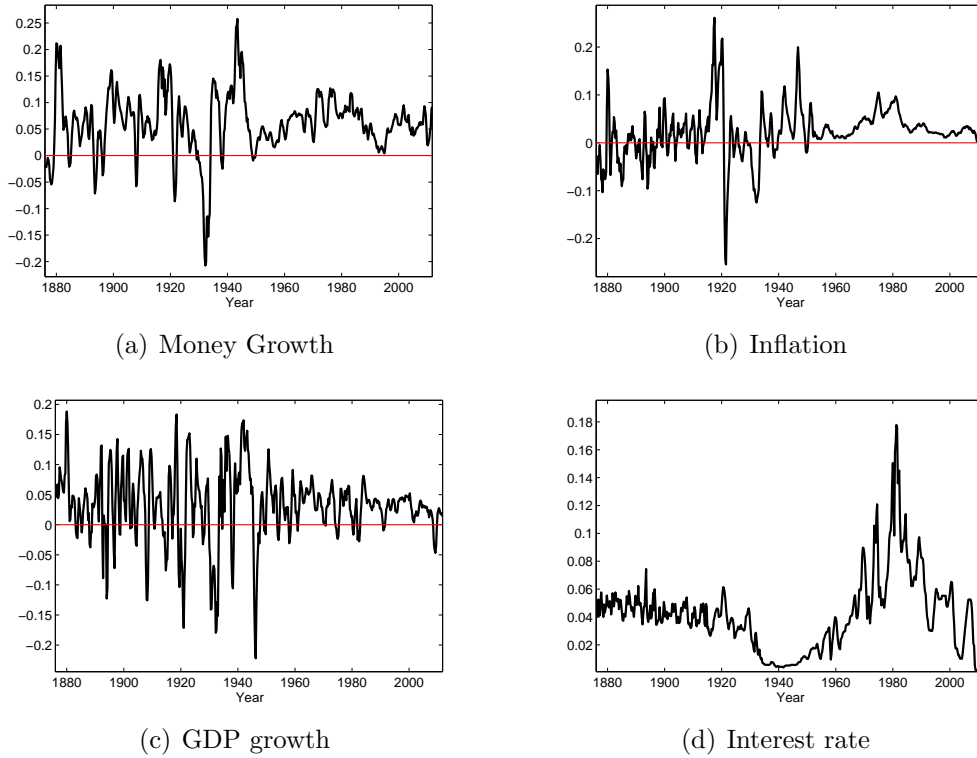


Figure 1: Data between 1876Q1 and 2011Q4.

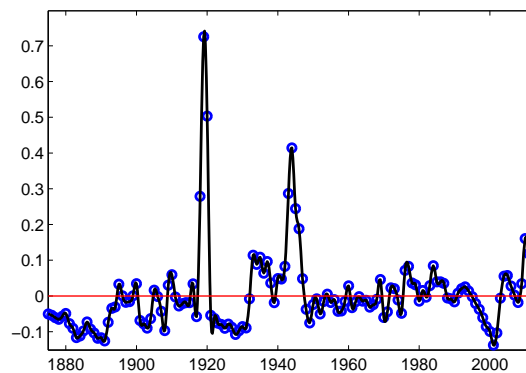


Figure 2: Primary deficits over debt between 1876Q1 and 2011Q4, the  $o$  indicate the original data by Bohn (2008) while the solid line indicates the final interpolated data.

In Figure 3, we plot the filtered time series which we use in Section 2 of the main text for the OLS regression. To keep this subsection self-contained, denote the unfiltered time series by  $x$  and the corresponding filtered time series by  $x(\beta)$ . The filter is defined as  $x(\beta)_t = \alpha \sum_{k=-n}^n \beta^{|k|} x_{t+k}$ , where  $\alpha = (1 - \beta)^2 / (1 - \beta^2 - 2\beta^{n+1}(1 - \beta))$  is chosen such that the sum of weights equals one. We set  $n$  to eight and choose  $\beta = 0.95$ , which achieves the intention of focusing on low-frequency variations.

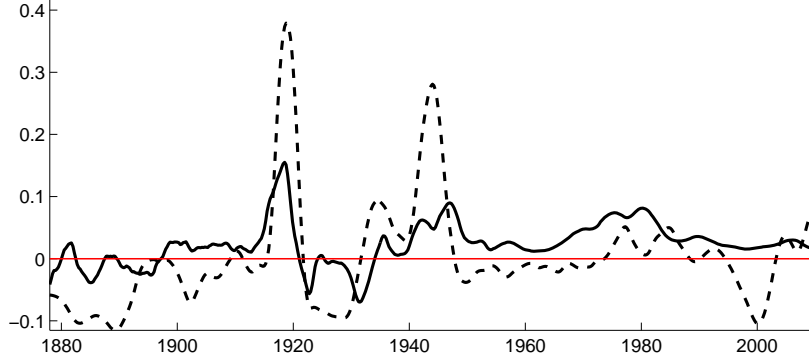


Figure 3: Filtered time series of inflation (solid) and deficit over debt (dashed).  $\beta = 0.95$

## 1.2 Metropolis-within-Gibbs sampler

We use the Metropolis-within-Gibbs sampler to obtain draws for the parameters of our time-varying VAR. The sampler is structured as follows. We start by initializing  $\mathbf{b}^T, \mathbf{H}^T, \mathbf{V}$  with estimates from our training sample. We then draw the VAR coefficients  $\mathbf{a}^T$  from  $p(\mathbf{a}^T | \mathbf{y}^T, \mathbf{b}^T, \mathbf{H}^T, \mathbf{V})$ , where the superscript  $T$  denotes the history of the variable (or vector of variables) up to time  $T$ . In the next step we draw the  $\mathbf{b}^T$  from  $p(\mathbf{b}^T | \mathbf{y}^T, \mathbf{a}^T, \mathbf{H}^T, \mathbf{V})$ . Up to this step, the our sampler is identical to the one described in Primiceri (2005). In the fourth step we deviate from the Gibbs sampler of Primiceri (2005) and draw the log volatilities using the Metropolis-Hastings algorithm suggested by Watanabe and Omori (2004). We hence draw  $\mathbf{H}^T$  from  $p(\mathbf{H}^T | \mathbf{y}^T, \mathbf{a}^T, \mathbf{b}^T, \mathbf{V})$ . In the last step we draw the variance-covariance matrix  $\mathbf{V}$ , by sampling  $\mathbf{Q}$  from  $p(\mathbf{Q} | \mathbf{y}^T, \mathbf{A}^T, \mathbf{B}^T, \mathbf{H}^T)$ ,  $\mathbf{W}$  from  $p(\mathbf{W} | \mathbf{y}^T, \mathbf{A}^T, \mathbf{B}^T, \mathbf{H}^T)$  and  $\mathbf{S}$  from  $p(\mathbf{S}_1 | \mathbf{y}^T, \mathbf{A}^T, \mathbf{B}^T, \mathbf{H}^T) \dots p(\mathbf{S}_{n-1} | \mathbf{y}^T, \mathbf{A}^T, \mathbf{B}^T, \mathbf{H}^T)$ . This last step is again identical to the one described in Primiceri (2005).

## 1.3 Convergence Checks

To check the convergence of our sampler, we have used visual inspections and numerical convergence diagnostics. The visual inspections illustrate how the parameters move through the parameter space, thereby allowing us to check whether the chain gets stuck in certain areas. To visualize the evolution of our parameters, we use running mean plots and trace plots. For lack of space, we present only running mean plots and trace plots for the trace of the variance covariance matrices  $\mathbf{Q}$ ,  $\mathbf{W}$ , and  $\mathbf{S}$ . As can be seen in Figure 4, and Figure 5 running mean plots and trace plots both show that the mean of the parameter values stabilize as the number of iterations increases and that the chains are mixing quite well.

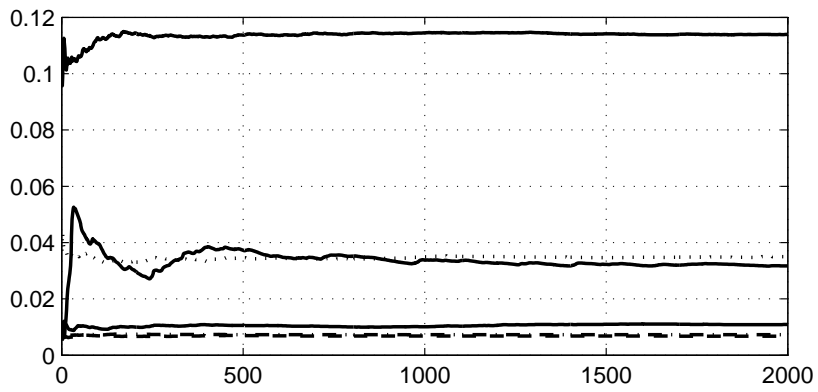


Figure 4: Running Mean Plot.

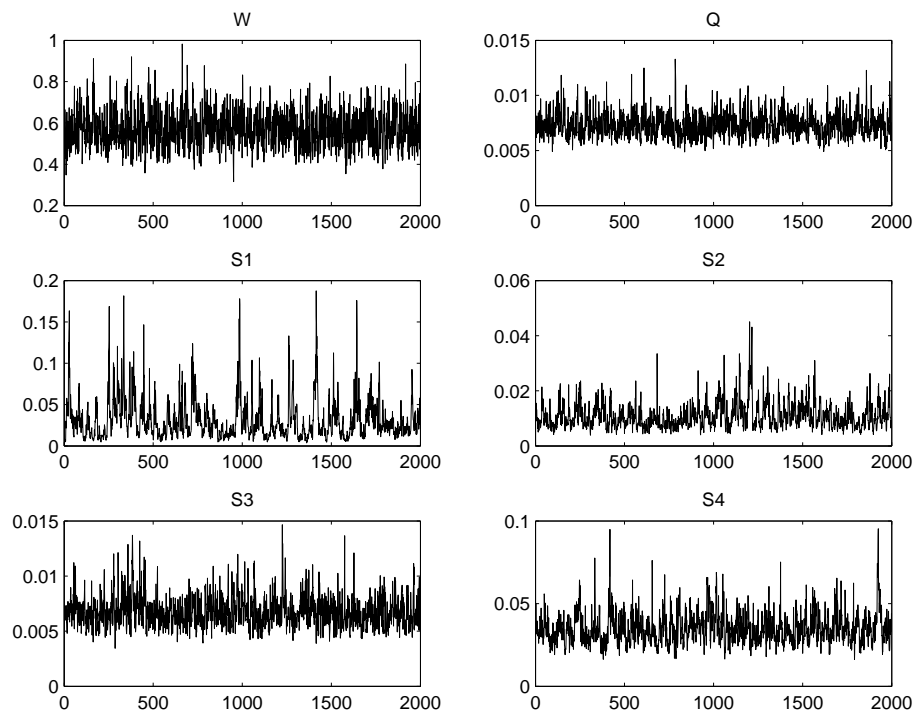
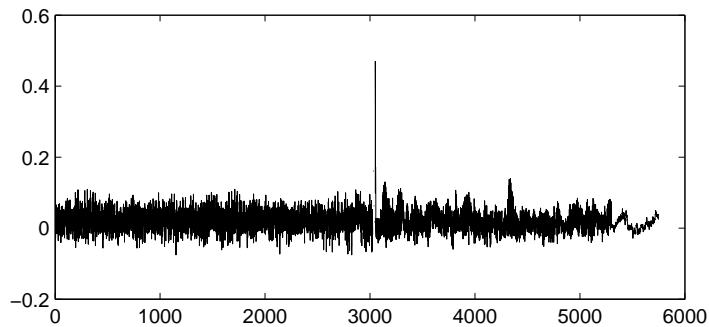
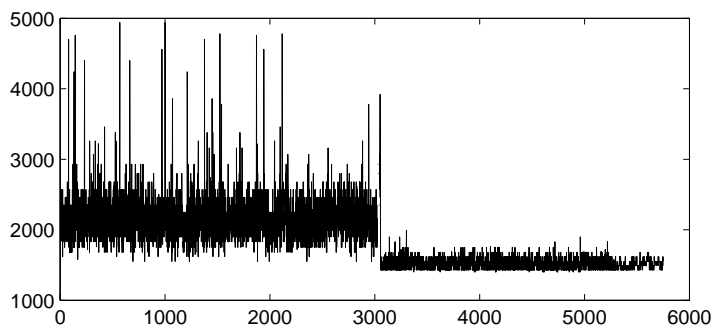


Figure 5: Trace Plot.

Additionally, we have calculated autocorrelations at the 10th lag as a numerical measure of the mixing characteristics of the Markov chain. High autocorrelations indicate a bad mixing of the chain that would exacerbate the convergence of the sampler. We have also computed the total number of draws needed to obtain a certain precision as suggested by Raftery and Lewis (1992).



(a) Autocorrelation at 10th draw.



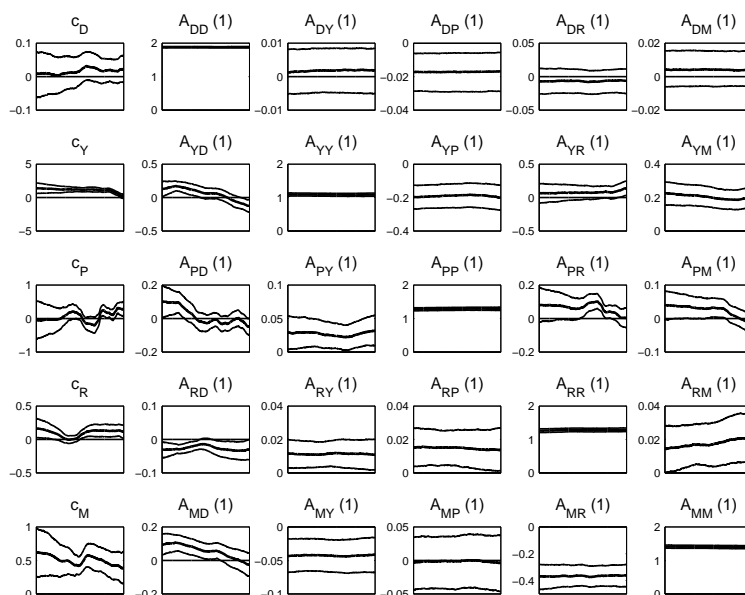
(b) Minimum Number of Draws.

Figure 6: Convergency diagnostics.

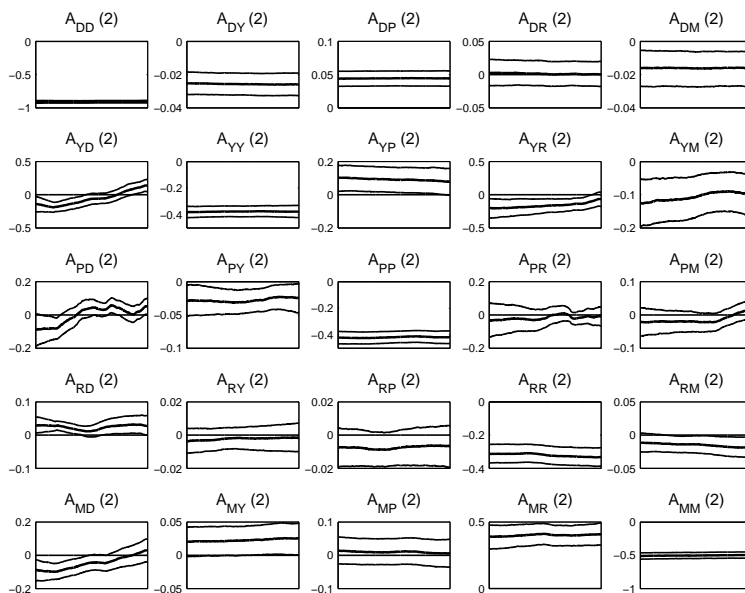
Figure 6 depicts the convergence diagnostics for all hyperparameters (points 1-3055), the stochastic volatilities (points 3056-5300) and the absolute maximum eigenvalue of the parameter matrix  $\mathbf{A}_t$  (points 5301-5749). As can be seen in Figure 6(a), most of the autocorrelations are below 0.1 indicating that the chain mixes quite well and that the sampler performs efficiently. Moreover, as can be seen in Figure 6(b), the number of draws suggested by the Raftery and Lewis (1992) diagnostic is far below our actual number of draws (we used 0.025 for the quantile, 0.025 for the level of precision, and the 0.95 for probability of obtaining the required precision). To summarize, according to convergence tests conducted the sampler seems to be converged.

## 1.4 Supplementary results

### 1.4.1 VAR coefficients



(a) Constant and AR(1) parameter



(b) AR(2) parameter

Figure 7: Time-varying parameter estimates of matrix  $\mathbf{A}$ .

### 1.4.2 Unconditional second moments

In the paper we mainly focus on the relationship between deficits and inflation at frequency zero. The additional results in this subsection present different measures of co-movements

between public deficits and inflation for different frequencies. To keep this subsection self-contained, we re-state the state-space system of the time-varying VAR model:

$$\begin{aligned}\mathbf{x}_{t+1} &= \mathbf{A}_{t|T}\mathbf{x}_t + \mathbf{B}_{t|T}\mathbf{w}_{t+1} \\ \mathbf{y}_{t+1} &= \mathbf{C}_{t|T}\mathbf{x}_t + \mathbf{D}_{t|T}\mathbf{w}_{t+1} ,\end{aligned}\tag{1}$$

where  $\mathbf{x}_t$  is the  $n_x \times 1$  state vector,  $\mathbf{y}_{t+1}$  is an  $n_y \times 1$  vector of observables,  $\mathbf{w}_{t+1}$  is an  $n_w \times 1$  Gaussian random vector with mean zero and unit covariance matrix that is distributed identically and independently across time. The matrices  $\mathbf{A}, \mathbf{B}, \mathbf{C}$ , and  $\mathbf{D}$  are functions of a vector of the time-varying structural model parameters. Given this representation the corresponding temporary spectral density at time  $\mathbf{t}$  of matrix  $Y$  is

$$S_{Y,t|T}(\omega) = \mathbf{C}_{t|T} (I - \mathbf{A}_{t|T}e^{-i\omega}) \mathbf{B}_{t|T}\mathbf{B}'_{t|T} (I - \mathbf{A}_{t|T}e^{-i\omega}) \mathbf{C}'_{t|T} + \mathbf{D}_{t|T}\mathbf{D}'_{t|T} .\tag{2}$$

The temporary spectral density matrix is a Fourier transformation of the sequence of temporary autocovariance matrices  $EY_{t|T}Y'_{t-j|T}$  at time  $\mathbf{t}$  which can be recovered via the formula:

$$EY_{t|T}Y'_{t-j|T} = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_{Y,t|T}(\omega) e^{i\omega j} d\omega\tag{3}$$

Given this temporary autocovariances, we calculate the unconditional second moments of the model variables. In particular, Figure 8 and Figure 9(a) present the implied temporary standard deviations of the different variables over time given the VAR model and the unconditional correlation between primary deficits over debt and inflation, respectively.

The correlation at different frequencies is calculated in the following way. Let  $y_t$  and  $x_t$  be two scalar components of  $Y_t$  and  $S_{y,t|T}(\omega)$  and  $S_{x,t|T}(\omega)$  the temporary spectral density functions of  $y_t$  and  $x_t$ , respectively. Finally, let  $C_{yx,t|T}(\omega)$  be the temporary co-spectrum between  $y_t$  and  $x_t$  at time  $\mathbf{t}$ . Given these definitions the temporary coherence (or coherency squared)  $\Upsilon_{yx,t|T}(\omega)$  between  $y_t$  and  $y_t$  at time  $\mathbf{t}$ , is defined as

$$\Upsilon_{yx,t|T}(\omega) = \frac{C_{yx,t|T}(\omega)^2 + Q_{yx,t|T}(\omega)^2}{S_{x,t|T}(\omega)S_{y,t|T}(\omega)} = \frac{|S_{yx,t|T}(\omega)|^2}{S_{x,t|T}(\omega)S_{y,t|T}(\omega)}\tag{4}$$

where  $S_{xy,t|T}(\omega)$  is the temporary cross-spectrum and  $Q_{xy,t|T}(\omega)$  the temporary quadrature spectrum. As mentioned in the paper, this measure can be interpreted as the  $R^2$  at the frequency  $\omega$  of the two time series, i.e. how much of the variation in each variable is explained by the joint variation. This measure is also related to the dynamic correlation between  $y_t$  and  $x_t$  as proposed by Croux, Forni, and Reichlin (2001). We follow Croux et al. (2001) and calculate the dynamic correlation, because coherency is not symmetric and involves complex numbers. More precisely, the dynamic correlation is defined as:

$$\rho_{xy,t|T}(\omega) = \frac{C_{xy,t|T}(\omega)}{\sqrt{S_{x,t|T}(\omega)S_{y,t|T}(\omega)}} .\tag{5}$$

The results for this temporary correlation at different frequencies are illustrated in Figure 9(b). For readability, we just present results for the time episode between 1960 and 2000.

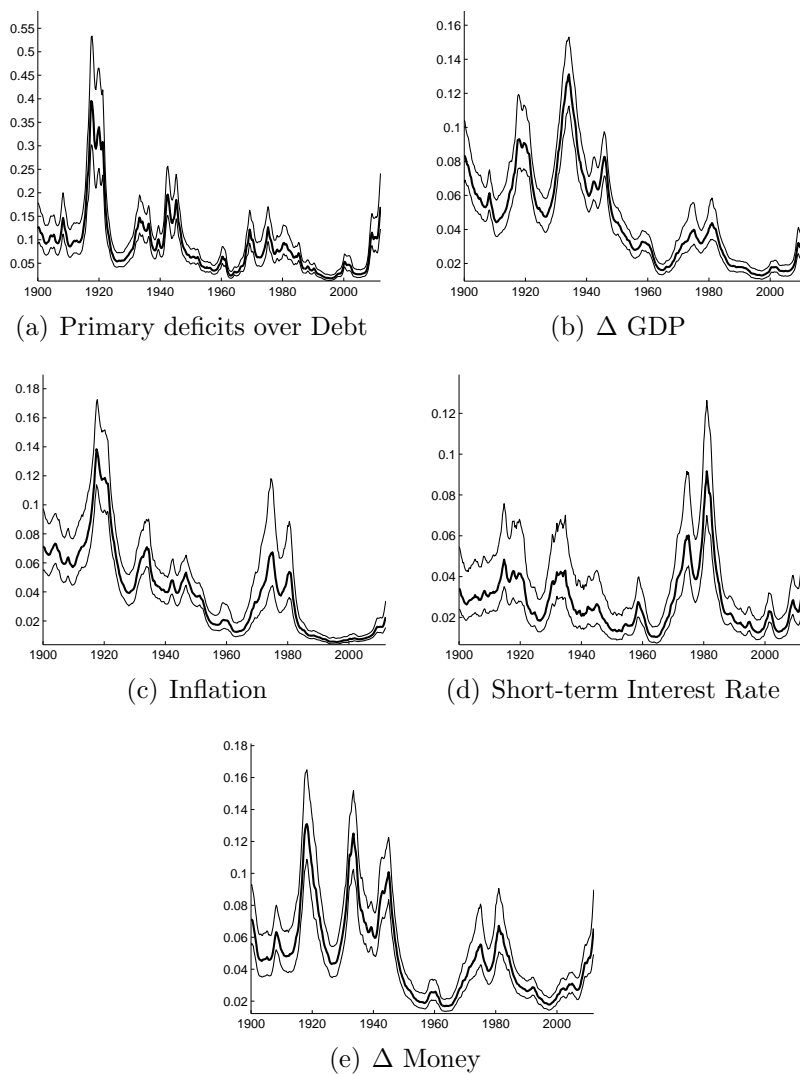


Figure 8: Standard deviations of the variables.



Both Figures 9(a) and 9(b) show that the correlation between public deficits and inflation was increasing during the 1960s with a all time high in the mid 1970s. After 1980 the correlation goes down again. Figure 9(b) points out the importance of low frequencies for the co-movements between deficits and inflation.

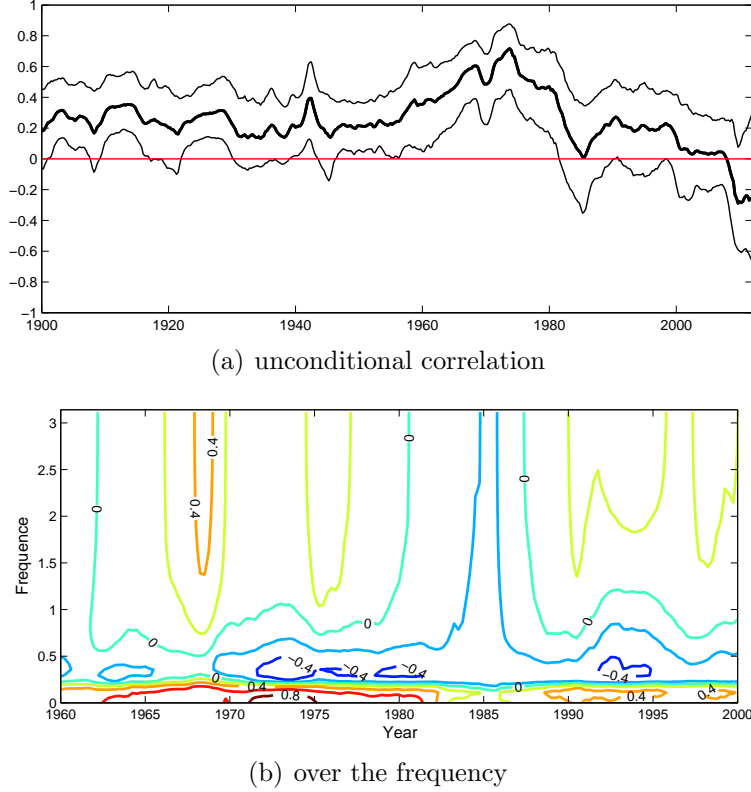


Figure 9: Correlation of primary deficits over debt with inflation

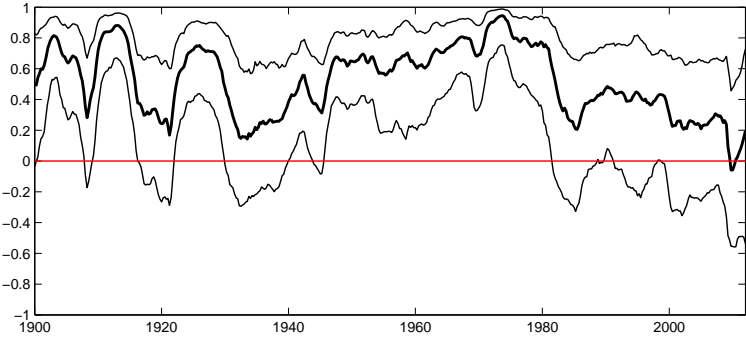
To further investigate the different frequencies, we compute the dynamic correlation for different frequency bands. In particular, we are interested in the co-movement of two time series in the long run and in the short run. While the first can be interpreted as frequencies longer than eight years, we define the latter one as a measure at the business cycles frequencies, e.g. periods between one and eight years. The temporary dynamic correlation on a frequency band is given by

$$\rho_{xy,t|T}(\omega^+) = \frac{\int_{\omega^+} C_{xy,t|T}(\omega) d\omega}{\sqrt{\int_{\omega^+} S_{x,t|T}(\omega) d\omega \int_{\omega^+} S_{y,t|T}(\omega) d\omega}}, \quad (6)$$

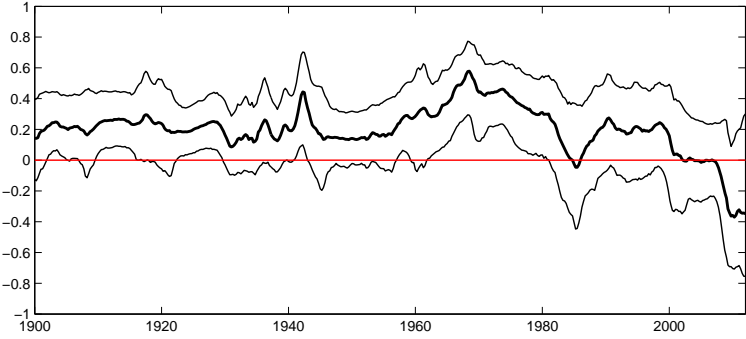
where  $\omega^+ = [0, \pi^*)$ ,  $0 \leq \pi^* \leq \pi$  and  $\omega^+ = [\pi^*, \pi)$ ,  $0 < \pi^* \leq \pi$  are related to the long-run dynamic correlation or the short-run dynamic correlation as defined above, respectively.

In comparison to Figure 9(a), the figures illustrate that for both frequency bands we can observe a similar pattern but, especially, for low-frequencies the changes are more striking. While, for the 1970s the correlation between public deficits and inflation is significant positive with values close to 0.9 in the long run, we find that the correlation for the time from 1980

onward becomes much smaller and most of the time insignificant different from zero for the long run as well as for business cycle frequencies.



(a) frequencies longer than eight years



(b) business cycles frequencies

Figure 10: Correlation of primary deficits over debt with inflation for different frequency bands.

## 2 Robustness

### 2.1 Alternative measures

In Section 2 of the corresponding paper we illustrate the low-frequency relationship between inflation and primary deficits over debt by presenting scatter plots of the corresponding filtered time series, where the filter is defined as  $x_t(\beta) = \alpha \sum_{k=-n}^n \beta^{|k|} x_{t+k}$ . Here,  $\alpha = (1 - \beta)^2 / (1 - \beta^2 - 2\beta^{k+1}(1 - \beta))$  is being chosen such that the sum of weights equals one. The number of leads and lags  $n$  is set to 8 and  $\beta = 0.95$ . The slope of the scatter plots is equal to the OLS estimate of the following regression

$$\pi_t(\beta) = const + b_f d_t(\beta) + error, \quad (7)$$

where we assume orthogonality between  $d_t(\beta)$  and the error term.

Alternatively, we can calculate the low-frequency relationship directly without filtering the data by employing the efficient lead/lag estimator postulated by Stock and Watson (1993). The corresponding regression formula using unfiltered data is given by

$$\pi_t = const + b_f d_t + \sum_{i=-n}^n \gamma_i \Delta d_{t-i} + error, \quad (8)$$

where  $b_f$  is the dynamic OLS estimator (DOLS). The number of leads and lags is chosen to be 8. Finally, we employ Newey-West HAC standard errors for both estimation approaches.

Column 2 and 3 of Table 1 present the estimation results for both regressions as well as for different sub-samples.

As third alternative, we estimate the low-frequency relationship by employing the method suggested by Sargent and Surico (2011). In particular, we estimate the VAR model and use its coefficients to compute the low-frequency relationship. The VAR model contains unfiltered data instead of filtered data. Hence, we follow Sargent and Surico (2011) and make use of one result provided by Whiteman (1984). In particular, Whiteman (1984) shows that for  $\beta$  close to 1, the regression coefficient in equation (7) can be approximated by the sum of lagged regression coefficients of a projection of  $\pi$  on  $d$ . Formally, define the projection as

$$\pi_t = \sum_{j=-\infty}^{\infty} \iota_j d_{t-j} + \epsilon_t, \quad (9)$$

with the orthogonality assumption  $E[d_{t-j}\epsilon_t] = 0$ . The regression coefficient is approximated as

$$b_f \approx \sum_{j=-\infty}^{\infty} \iota_j \quad (10)$$

Sargent (1987) shows that the sum of lagged regression coefficients is equal to the cross

Sample	OLS	DOLS	BVAR(2)	BVAR(4)
1900-1933	0.2882 (0.0499)	0.2879 (0.0570)	0.6305 [0.2912;0.9869]	0.0496 [-0.3029;0.3900]
1934-1951	0.0909 (0.0350)	0.2999 (0.1585)	0.0579 [-0.1755;0.2351]	0.2686 [0.1938;0.3427]
1952-1983	0.8076 (0.1214)	1.0604 (0.1108)	1.3378 [1.0984;1.6459]	1.2013 [0.9369;1.5282]
1984-2009	0.0691 (0.0242)	0.0913 (0.0913)	0.1418 [0.0881;0.2101]	0.2015 [0.1104;0.3008]
1900-2009	0.2212 (0.0395)	0.2455 (0.0791)	0.4275 [ 0.2251;0.6223]	0.2354 [0.0459;0.4421]

Table 1: Low-frequency relationship between primary deficits over debt and inflation for different subsamples calculated from OLS estimates of filtered time series and from DOLS and BVAR model estimates of unfiltered time series. The values in parenthesis are Newey-West HAC standard errors, values in brackets correspond to 16% and 84% probability bands.

spectrum of  $\pi$  and  $d$ ,  $S_{\pi d}$ , divided by the spectrum of  $d$ ,  $(S_d)$ , at frequency zero:

$$\sum_{j=-\infty}^{\infty} \iota_j = \frac{S_{\pi d}(0)}{S_d(0)} \quad (11)$$

Column 4 and 5 of Table 1 show the estimation results for  $b_f$  based on time-invariant Bayesian VAR models with 2 and 4 lags for different sub-samples. Each BVAR model includes primary deficits over debt, output growth, inflation, nominal interest rate, and money growth. For estimation, we assume a weak Normal-Whishart prior for the coefficients and the covariance matrix of the BVAR model. Afterwards, we draw parameter vectors from the posterior of the BVAR model and retain those draws for which stationarity of the associated VAR model is ensured. Finally, we calculate  $b_f$  for each of the 1000 posterior draws. Figure 11 illustrates the similar distribution of  $b_f$  with respect to different lag specifications. To conclude, all aforementioned methodologies show similar patterns. In particular, we find a low-frequency relationship for the time between 1952 and 1983, but not for the period from 1984 onward.

Finally, we also calculate time-varying estimates of equation (7) and (8) by employing a rolling sample with a fixed window length of 120 quarters. Figure 12 presents the time-varying estimates of  $b_f$ . The results of both time-varying estimation approaches are similar and indeed comparable to our main result.

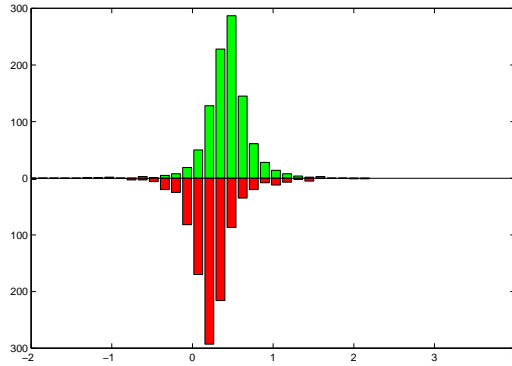
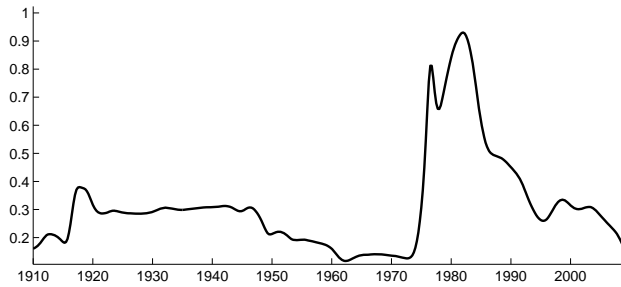
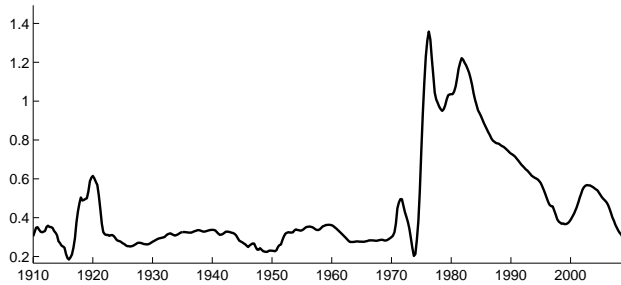


Figure 11: Implied distribution of  $b_f$  for BVAR(2) - green bars - and BVAR(4) - red bars - using sample 1900:q1-2009:q4.



(a) OLS estimate



(b) DOLS estimate

Figure 12: Rolling sample (fixed window) regression coefficients.

## 2.2 Further low-frequency relationships

Below we analyze whether the low-frequency relationship between inflation and the primary deficits over debt ratio diminishes or even cancels out other well established low-frequency relationships. More precisely, we investigate the low-frequency relationship between inflation and money and between money and interest rates as postulated by Lucas (1980) and recently investigated by Sargent and Surico (2011). As Figures 13(a) and 13(c) show, we obtain results similar to those of Sargent and Surico (2011), i.e. our finding of an additional positive relationship does not crowd out the existing relationships.

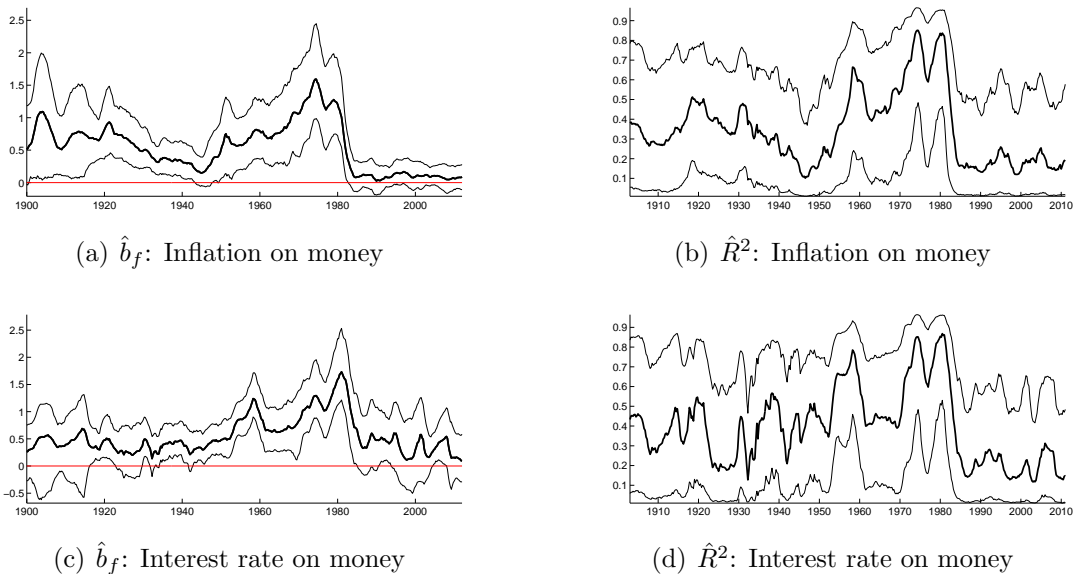


Figure 13: Selected low-frequency relationships.

## 2.3 Alternative TVP-VAR Specifications

In the following, we describe different robustness checks which we employ to investigate the sensitivity of our results. First, we change the interpolation method for the primary deficits over debt time series. In particular, we employ the methods proposed by Chow and Lin (1971) and Litterman (1983) next to the cubic-spline approach. Figure 14 presents the interpolated time series.

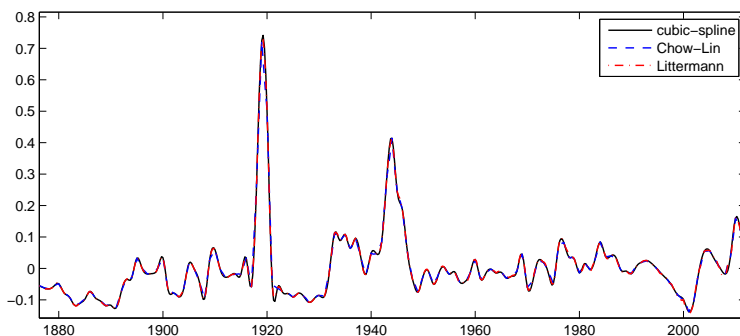


Figure 14: Interpolated time series for primary deficits over debt using different interpolation methods.

We use for both methods, Chow and Lin (1971) and Litterman (1983), as related time series for the interpolation real GDP and the Price index as described in Section 3 of the main paper. The results for all methods are quite similar. We decide to use the interpolated time series based on the cubic-spline method for our baseline estimation. This is based on the fact that next to the time series employed in the VAR model, we have no other suitable long time series available whose information can be exploited to interpolate the primary deficit-over debt time series. But this is necessary for the application of the methods proposed by Chow and Lin (1971) and Litterman (1983). Using the same time series for interpolation and estimation of the TVP-VAR would imply that we use the data twice. Therefore, we only show that the interpolated time series are similar, but do not employ the different interpolated time series in the estimation.

While the theory about inflationary consequences of public deficits (see, e.g., Sims, 2011) highlights the role of market values, unfortunately, the long time series for government debt by Bohn (2008) is only available in par values and not in market values. However, since we are interested in the low-frequency relationship, temporary differences between market and par values should not be critical (see also Bohn, 1991). Nevertheless, we analyze the robustness of our results with respect to market value of debt by constructing a quarterly proxy. In particular, we calculate quarterly primary deficits calculated from NIPA data and market value of privately held gross federal debt calculated by the Federal Reserve Bank of Dallas. The final quarterly time series covers the time from 1947:q1 until 2010:q1. Figure 15 illustrates that our main result is robust with respect to differences between market and par values of debt.

What is more, since our analysis is based on a long time series of the GDP deflator, we want to investigate how robust the results are regarding different measures of inflation. To this end, we substitute the GDP deflator with the CPI deflator, which is the time series

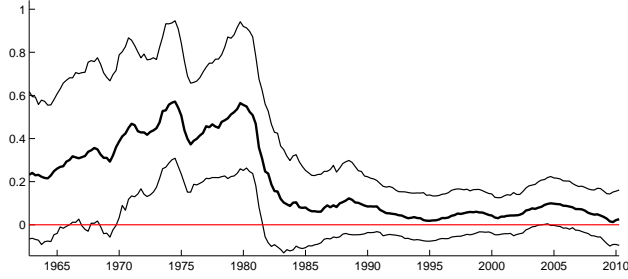


Figure 15:  $\hat{b}_f$ : Median and 68% central posterior bands for the time-varying regression coefficient inflation on primary deficits over debt. Robustness check with quarterly primary deficits over market value of debt.

‘Consumer Price Index for All Urban Consumers (CPIAUCSL)’ taken from the FRED II database of the Federal Reserve Bank of St. Louis. Similarly to the above mentioned marked value of debt this time series is just available from 1947:q1 onward. Hence, we run the estimation with a shorter sample starting in 1948:q1. Figure 16 shows the main result. For both robustness checks which deal with shorter samples we employ also a shorter training sample at the beginning of the observations. This gives us also confidence that estimation results are not sensitive regarding our choice of the training sample.

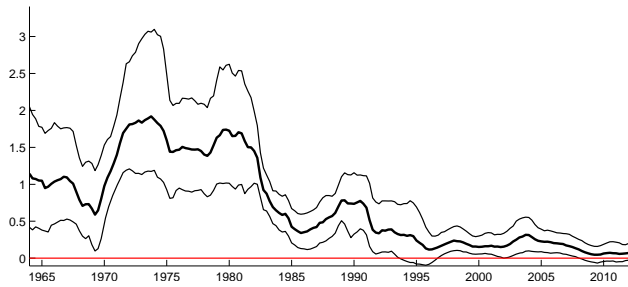


Figure 16:  $\hat{b}_f$ : Median and 68% central posterior bands for the time-varying regression coefficient inflation on primary deficits over debt. Robustness check CPI inflation instead of GDP deflator.

Next, we check the robustness of our result with respect to other interest rates measures and another measure of fiscal stance. Moreover, we apply a different ordering of the variables in the VAR model. In particular, the alternative ordering is: primary deficits over debt, money growth, inflation, interest rate, and GDP growth. Figures 17 to 19 show the main result based on these different VAR specifications. While different interest rates have almost no impact on our result, the change of the fiscal variable also changes the estimated relationship slightly. Also, our main finding of an high relationship in the 1970s, which deteriorates after 1980, still exists.



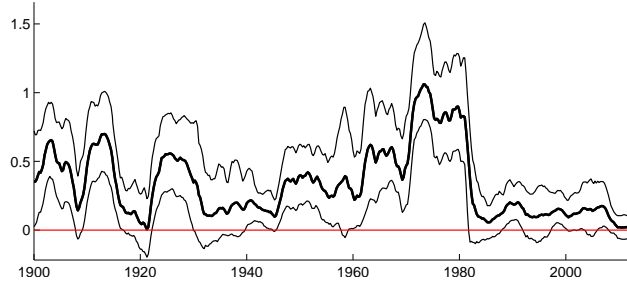


Figure 17:  $\hat{b}_f$ : Median and 68% central posterior bands for the time-varying regression coefficient inflation on primary deficits over debt. Robustness check with 3m nominal interest rates instead of 6m interest rates.

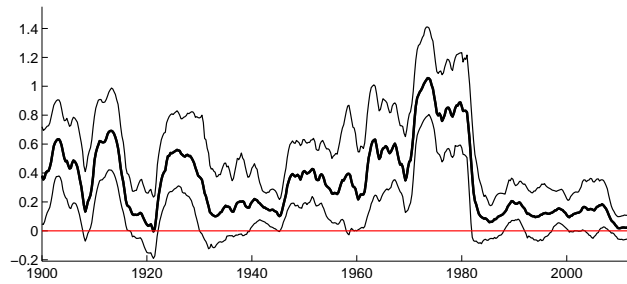


Figure 18:  $\hat{b}_f$ : Median and 68% central posterior bands for the time-varying regression coefficient inflation on primary deficits over debt. Robustness check with 3m real interest rates instead of 6m interest rates.

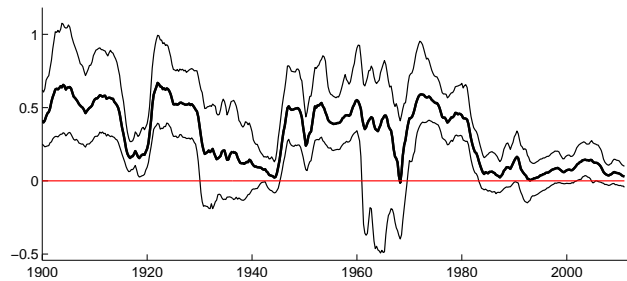


Figure 19:  $\hat{b}_f$ : Median and 68% central posterior bands for the time-varying regression coefficient inflation on debt growth. Robustness check with real debt growth instead of primary deficits over debt.

## 2.4 Counterfactuals

In the following section we provide additional results regarding the counterfactual exercise in Section 4.3 of the paper. First, we present in addition to the unconditional median of  $\hat{b}_f$  also the corresponding 68% central posterior bands. Figure 20 shows the results with posterior bands related to Figures 8 and 9 of the main paper. The dashed line represents the median of  $\hat{b}_f$  without fixing the VAR model coefficients.

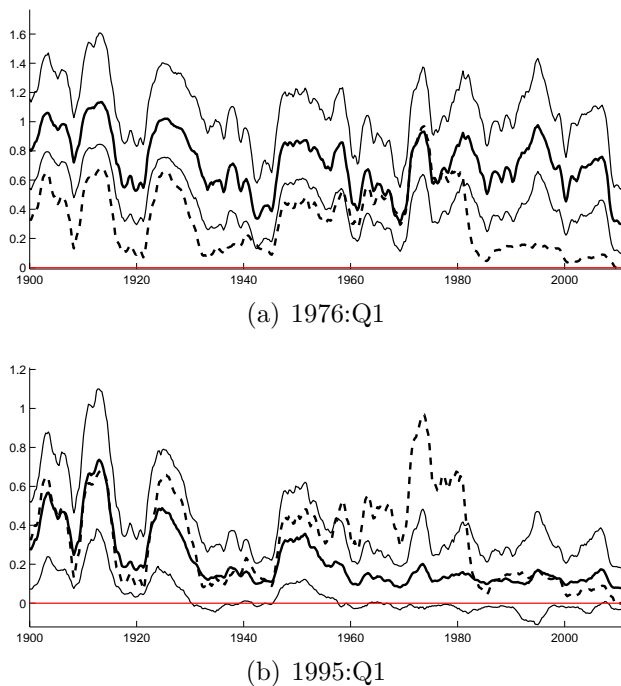
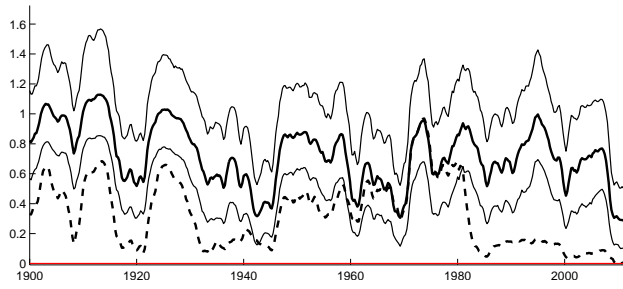


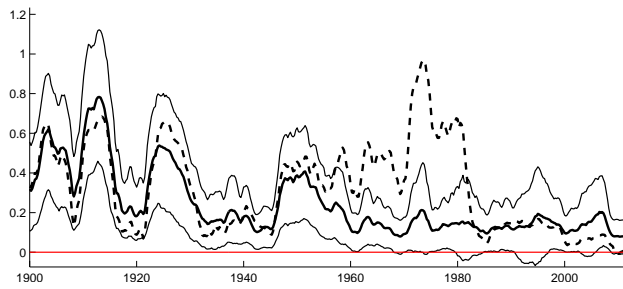
Figure 20: Counterfactual experiment: Median and 68% central posterior bands for  $\hat{b}_f$  for VAR model coefficients (**A** and **B**) fixed at different point in times. The dashed line represents the median of  $\hat{b}_f$  without fixing the VAR model coefficients.

In the following we investigate how sensitive these results are with respect to the chosen points in time. Therefore, we run the counterfactual experiment by fixing the matrices **A** and **B** to the mean over a time span instead by fixing both to particular quarters. Figure 21 shows the results for two different time spans, first, between 1970:Q1 and 1978:Q4 and, second, between 1985:Q1 and 1994:Q1. Remarkably, these figures illustrate the robustness of our counterfactual analysis regarding the chosen points in time.

Finally, we run a counterfactual experiment without choosing any specific point in time. In particular, we scale the matrix **H** for each point in time to 1. This means that in each point in time the shocks which hit the economy have the same size. The remaining matrices **A** and **B** are drawn from their posterior distribution at each point in time. Figure 22 confirms our finding that the movements of the low-frequency relationship are note due to changes in the volatilities of the shocks but driven by changes in the systematic behavior of the economy.



(a) 1970:Q1-1978:Q4



(b) 1985:Q1-1994:Q4

Figure 21: Counterfactual experiment: Median and 68% central posterior bands for  $\hat{b}_f$  for VAR model coefficients (**A** and **B**) fixed at mean over different time spans. The dashed line represents the median of  $\hat{b}_f$  without fixing the VAR model coefficients.

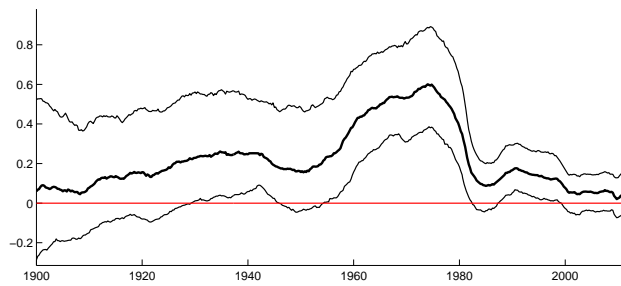


Figure 22: Counterfactual experiment: Median and 68% central posterior bands for  $\hat{b}_f$  with matrix **H** scaled over the complete time span.

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