Online Supplementary Appendix for "Social Interactions and Social Preferences in Social Networks"

Chih-Sheng Hsieh*
Department of Economics
National Taiwan University

Xu Lin†
Department of Economics
Virginia Tech

November 20, 2020

*Taipei 10617, Taiwan. Tel: (+886) 233668326, Email: cshsieh@ntu.edu.tw.
†3016 Pamplin Hall, 880 West Campus Drive, Blacksburg, VA 24061. Tel: 1-540-2319636 Email: xulin88@vt.edu.

A Conditional posterior distributions

Based on the joint model of Equations (14) and (17), we apply the Gibbs sampling to simulate draws from the posterior distribution in Equation (22). Here we list the set of derived conditional posterior distributions that compose the Gibbs sampler:

(i-1) $P(z_{i,g}|Y_g, W_g, \theta, \tau_g, Z_{-i,g})$, $i = 1, \cdots, m_g$, $g = 1, \cdots, G$.

By applying Bayes' theorem,

$$P(z_{i,g}|Y_g, W_g, \theta, \tau_g, Z_{-i,g}) \propto \phi_{\bar{d}}(z_{i,g}; \mu_{z,g}, I_{\bar{d}}) \cdot P(Y_g, W_g|\theta, \tau_g, Z_g), \tag{A.1}$$

where $\phi_{\bar{d}}(.; \nu_{z,g}, I_{\bar{d}})$ is the multivariate normal density function. We simulate $z_{i,g}$ from Equation (A.1) using the Metropolis-Hastings (M-H) algorithm.

(i-2) $P(\mu_{z,g}|Y_g, W_g, \theta, \tau_g, Z_g)$, $g = 1, \cdots, G$.

By applying Bayes' theorem,

$$P(\mu_{z,g}|Y_g, W_g, \theta, \tau_g, Z_g) \propto \mathscr{N}_{\bar{d}}\left(\frac{m_g \bar{Z}_g}{m_g + 1/\xi^2}, \frac{1}{m_g + 1/\xi^2}\right), \tag{A.2}$$

where $\bar{Z}_g = \frac{1}{m_g}\sum_{i=1}^{m_g} z_{i,g}$. We simulate $\mu_{z,g}$ directly from Equation (A.2).

(ii) $P(\omega|\{W_g\}, \{Z_g\})$.

By applying Bayes' theorem, we have

$$P(\omega|\{W_g\}, \{Z_g\}) \propto \phi_{G+R+\bar{d}}(\omega; \omega_0, \Omega_0) \cdot \prod_{g=1}^{G} P(W_g|Z_g, \omega) \cdot I(\omega \in O_1), \tag{A.3}$$

where $I(A)$ is an indicator function with $I(A) = 1$ if A holds and $I(A) = 0$ otherwise. We simulate ω from Equation (A.3) using the M-H algorithm.

(iii) $P(\Lambda|\{Y_g\}, \{W_g\}, \beta, \sigma_v^2, \delta, \{\tau_g\}, \{Z_g\})$.

By applying Bayes' theorem, we have

$$P(\Lambda|\{Y_g\}, \{W_g\}, \beta, \sigma_v^2, \delta, \{\tau_g\}, \{Z_g\}) \propto \prod_{g=1}^{G} P(Y_g|W_g, \Lambda, \beta, \sigma_v^2, \delta, \tau_g, Z_g) \cdot I(\Lambda \in O_2). \tag{A.4}$$

We simulate Λ from Equation (A.4) using the M-H algorithm.

(iv) $P(\beta|\{Y_g\}, \{W_g\}, \Lambda, \sigma_v^2, \delta, \{\tau_g\}, \{Z_g\})$.

By applying Bayes' theorem, we have

$$P(\beta|\{Y_g\}, \{W_g\}, \Lambda, \sigma_v^2, \delta, \{\tau_g\}, \{Z_g\}) \propto \phi_{2k+1}(\beta; \beta_0, B_0) \cdot \prod_{g=1}^{G} P(Y_g|W_g, \Lambda, \beta, \sigma_v^2, \delta, \tau_g, Z_g).$$

Since both $\phi_{2k+1}(\beta;\beta_0,B_0)$ and $P(Y_g|W_g,\Lambda,\beta,\sigma_v^2,\delta,\tau_g,Z_g)$ are normal density functions, we can simplify the expression to

$$
\begin{aligned}
&P(\beta|\{Y_g\},\{W_g\},\Lambda,\sigma_v^2,\delta,\{\tau_g\},\{Z_g\}) \propto \mathscr{N}_{2k+1}\left(\beta;\hat{\beta},\mathbf{B}\right)\\
&\hat{\beta} = \mathbf{B}\left(B_0^{-1}\beta_0 + \sum_{g=1}^{G}\mathbf{X}_g'(\sigma_v^2 I_{m_g})^{-1}(S_gY_g - \mathbf{Z}_g\delta - l_g\tau_g)\right)\\
&\mathbf{B} = \left(B_0^{-1} + \sum_{g=1}^{G}\mathbf{X}_g'(\sigma_v^2 I_{m_g})^{-1}\mathbf{X}_g\right)^{-1},
\end{aligned}
\tag{A.5}
$$

where $\mathbf{X}_g = (W_g^{\mathrm{T}}l_g, X_g, W_gX_g)$, $\mathbf{Z}_g = (Z_g, W_gZ_g)$, and $S_g = (I_{m_g} - \lambda W_g - \lambda^{\mathrm{I}}W_g^{\mathrm{T}})$.

(v) $P(\sigma_v^2|\{Y_g\},\{W_g\},\Lambda,\beta,\delta,\{\tau_g\},\{Z_g\})$.

By applying Bayes' theorem, we have

$$
\begin{aligned}
&P(\sigma_v^2|\{Y_g\},\{W_g\},\Lambda,\beta,\delta,\{\tau_g\},\{Z_g\})\\
&\qquad\propto \mathscr{IG}\left(\sigma_v^2;\frac{\nu_0}{2},\frac{\varsigma_0}{2}\right)\prod_{g=1}^{G}P(Y_g|W_g,\Lambda,\beta,\sigma_v^2,\delta,\tau_g,Z_g)\\
&\qquad\propto \mathscr{IG}\left(\sigma_v^2;\frac{\nu_0+\sum_{g=1}^{G}m_g}{2},\frac{\varsigma_0+\sum_{g=1}^{G}v_g'v_g}{2}\right),
\end{aligned}
\tag{A.6}
$$

where $v_g = S_gY_g - \mathbf{X}_g\beta - \mathbf{Z}_g\delta - l_g\tau_g$.

(vi) $P(\delta|\{Y_g\},\{W_g\},\Lambda,\beta,\sigma_v^2,\{\tau_g\},\{Z_g\})$.

By applying Bayes' theorem, we have

$$
\begin{aligned}
&P(\delta|\{Y_g\},\{W_g\},\Lambda,\beta,\sigma_v^2,\{\tau_g\},\{Z_g\})\\
&\qquad\propto \mathscr{N}_{2\bar{d}}(\delta;\delta_0,\Delta_0)\prod_{g=1}^{G}P(Y_g|W_g,\Lambda,\beta,\sigma_v^2,\delta,\tau_g,Z_g),
\end{aligned}
$$

Similar to (v), we can further obtain

$$
\begin{aligned}
&P(\delta|\{Y_g\},\{W_g\},\Lambda,\beta,\sigma_v^2,\{\tau_g\},\{Z_g\}) \propto \phi_{2\bar{d}}(\delta;\hat{\delta},\mathbf{D}),\\
&\hat{\delta} = \mathbf{D}\left(\Delta_0^{-1}\delta_0 + \sum_{g=1}^{G}\mathbf{Z}_g'(\sigma_v^2 I_{m_g})^{-1}(S_gY_g - \mathbf{X}_g\beta - l_g\tau_g)\right)\\
&\mathbf{D} = \left(\Delta_0^{-1} + \sum_{g=1}^{G}\mathbf{Z}_g'(\sigma_v^2 I_{m_g})^{-1}\mathbf{Z}_g\right)^{-1},
\end{aligned}
\tag{A.7}
$$

(vii) $P(\tau_g|Y_g,W_g,\lambda,\beta,\sigma_u^2,\delta,Z_g)$, $g = 1,\cdots,G$.

By applying Bayes' theorem, we have

$$
P(\tau_g|Y_g,W_g,\Lambda,\beta,\sigma_v^2,\delta,Z_g) \propto \phi(\tau_g;\tau_0,T_0)\cdot P(Y_g|W_g,\lambda,\beta,\sigma_u^2,\delta,\tau_g,Z_g).
$$

Similar to (v), we can further obtain

$$\begin{aligned}
&P(\tau_g|Y_g, W_g, \Lambda, \beta, \sigma_v^2, \delta, Z_g) \propto \mathcal{N}(\tau_g; \hat{\tau}_g, R_g), \\
&\hat{\tau}_g = R_g\Big(T_0^{-1}\tau_0 + l_g'(\sigma_v^2 I_{m_g})^{-1}(S_g Y_g - \mathbf{X}_g\beta - \mathbf{Z}_g\delta)\Big), \\
&R_g = \Big(T_0^{-1} + l_g(\sigma_v^2 I_{m_g})^{-1} l_g'\Big)^{-1}.
\end{aligned} \tag{A.8}$$

When considering the model based on Equation (16) and Equation (19), we need additional prior assumptions on the latent variables A_g and their coefficients γ_2, γ_3 in the network formation model and δ_3, δ_4 in the outcome equation, as well as the reciprocity coefficient ρ. Accordingly, we add and update some of the prior distributions in Equation (21) as follows:

$$\begin{aligned}
a_{i,g} &\sim U[-1,1], \quad i = 1, \cdots, m_g; \ g = 1, \cdots, G, \\
\omega = (\gamma', \zeta') &\sim \mathcal{N}_{G+R+2+\bar{d}}(\omega_0, \Omega_0) \text{ on the support } O_1, \\
\Lambda' = (\lambda, \rho) &\sim U_2(O_2'), \\
\delta = (\delta_1', \delta_2', \delta_3, \delta_4) &\sim \mathcal{N}_{2\bar{d}+2}(\delta_0, D_0),
\end{aligned} \tag{A.9}$$

where O_1 is discussed in the main text and O_2' specifies a space such that $I_{m_g} - \lambda(W_g + H_g(A_g, W_g, \rho))$ is invertible for all g. Since O_2' depends not only on data W_g's but also on unknown latent variables A_g which need to be estimated, the prior of Λ' is an "empirical Bayes prior" (Casella, 1985).

B Bayesian Model Comparison by Bayes Factor

we consider a goodness-of-fit comparison between the conventional social interactions model and the proposed altruistic social interactions model to examine whether adding altruism into the conventional social interactions model improves the fit of the model to the data. Since our models are estimated by the Bayesian MCMC sampling approach, the corresponding goodness-of-fit comparison is conducted based on the Bayes factor.

Recall that the conventional social interactions model is given by

$$Y_g = \lambda W_g Y_g + X_g\beta_1 + W_g X_g \beta_2 + l_g \tau_g + \epsilon_g, \quad g = 1, \cdots, G. \tag{B.1}$$

By introducing the altruistic preference and the direct externality effect, we obtain the altruistic social interactions model which is given by

$$Y_g = \lambda W_g Y_g + \lambda^{\mathrm{I}} W_g^{\mathrm{T}} Y_g + \eta^{\mathrm{I}} W_g^{\mathrm{T}} l_g + X_g\beta_1 + W_g X_g \beta_2 + l_g \tau_g + \epsilon_g, \quad g = 1, \cdots, G. \tag{B.2}$$

Let's call the altruistic social interactions model M_1 and the conventional social interactions model M_2. It is clear that M_2 is a nested model of M_1 under the parameter constraint: $\lambda^{\mathrm{I}} = \eta^{\mathrm{I}} = 0$. Without loss of generality, we ignore the issue of network endogeneity in both Equations (B.1) and (B.2) and only aim to evaluate the difference brought by altruism. When a network formation model like Equation (14) in the main text is introduced to correct the network endogeneity problem, it applies to both models M_1 and M_2 and therefore we expect it will not change the Bayesian model comparison result that we perform here.

Denote the prior probability of each model by $\underline{\pi}_j \equiv P(M_j) > 0$ and the posterior probability of model by $\bar{\pi}_j \equiv P(M_j|\{Y_g\}) = \frac{P(\{Y_g\}|M_j)\underline{\pi}_j}{P(\{Y_g\})}$, for $j = 1, 2$, where $P(\{Y_g\}|M_j) = \int P(\theta_j|M_j)P(\{Y_g\}|\theta_j, M_j)d\theta_j$ stands for the *marginal likelihood* under model M_j. To compare the goodness-of-fit of the two competing models, the posterior odds ratio is calculated as

$$\underbrace{\frac{\bar{\pi}_2}{\bar{\pi}_1}}_{\textit{Posterior odds}} = \underbrace{\frac{P(\{Y_g\}|M_1)}{P(\{Y_g\}|M_2)}}_{\textit{Bayes factor}} \times \underbrace{\frac{\underline{\pi}_2}{\underline{\pi}_1}}_{\textit{Prior odds}} . \tag{B.3}$$

In Equation (B.3), the prior odds ratio is usually set to 1 as the same prior probabilities are assumed for competing models. Therefore, the posterior odds ratio reduces to the Bayes factor.

We follow Chib (1995) and Chib and Jeliazkov (2001) to calculate the marginal likelihoods of M_1 and M_2 based on the model estimation MCMC sampling outputs. Note that

$$P(\{Y_g\}, \theta_j|M_j) = P(\{Y_g\}|\theta_j, M_j)P(\theta_j|M_j) = P(\theta_j|\{Y_g\}, M_j)P(\{Y_g\}|M_j),$$

and thus,

$$P(\{Y_g\}|M_j) = \frac{P(Y_g|\theta_j, M_j)P(\theta_j|M_j)}{P(\theta_j|\{Y_g\}, M_j)}.$$

After taking logarithm on both sides,

$$\ln P(\{Y_g\}|M_j) = \ln P(\{Y_g\}|\theta_j, M_j) + \ln P(\theta_j|M_j) - \ln P(\theta_j|\{Y_g\}, M_j). \tag{B.4}$$

Notice that Equation (B.4) holds for any θ_j and therefore we can evaluate Equation (B.4) at the posterior mean $\hat{\theta}_j$. The first and the second terms in Equation (B.4) are the log likelihood of model M_j and the log prior evaluated at $\hat{\theta}_j$, which are readily available. We use MCMC sampling outputs to calculate the third term. As an illustration, let us consider model M_1. The parameter vector θ_1 can be properly grouped into four blocks, which are $\Lambda = (\lambda, \lambda^{\mathrm{I}})$, $\beta = (\eta^{\mathrm{I}}, \beta_1', \beta_2')$, σ_ϵ^2, and $\{\tau_g\}$. Therefore,

$$\begin{aligned}&\ln P(\hat{\theta}_1|\{Y_g\}, M_1) = \\ &\quad \ln P(\hat{\Lambda}|\{Y_g\}, M_1) + \ln P(\hat{\beta}|\hat{\Lambda}, \{Y_g\}, M_1) + \ln P(\hat{\sigma}_\epsilon^2|\hat{\Lambda}, \hat{\beta}, \{Y_g\}, M_1) + \ln P(\{\hat{\tau}_g\}|\hat{\Lambda}, \hat{\beta}, \hat{\sigma}_\epsilon^2, \{Y_g\}, M_1).\end{aligned} \tag{B.5}$$

Now we discuss how to calculate each term in Equation (B.5).

Calculating $P(\hat{\Lambda}|\{Y_g\}, M_1)$ For the first term in Equation (B.5), as Λ is drawn using the M-H algorithm, we follow Chib and Jeliazkov (2001) to use the M-H outputs to calculate $\ln P(\hat{\Lambda}|\{Y_g\}, M_1)$. The M-H acceptance probability for Λ is

$$\alpha(\Lambda^*, \Lambda|\xi_1, \{Y_g\}, M_1) = \min\left\{1, \frac{f(\{Y_g\}|\Lambda^*, \xi_1, M_1)\pi(\Lambda^*)}{f(\{Y_g\}|\Lambda, \xi_1, M_1)\pi(\Lambda)} \frac{q(\Lambda|\Lambda^*)}{q(\Lambda^*|\Lambda)}\right\},$$

where $q(\Lambda^*|\Lambda)$ denotes the proposal density from Λ to Λ^* and ξ_1 stands for θ_1 excluding Λ. If we write $P(\Lambda^*, \Lambda|\xi_1, \{Y_g\}, M_1) = \alpha(\Lambda^*, \Lambda|\xi_1, \{Y_g\}, M_1)q(\Lambda^*|\Lambda)$ for the transition probability from Λ to Λ^*, then by reversibility of the M-H algorithm, we can write for any $\hat{\Lambda}$,

$$P(\hat{\Lambda}, \Lambda|\xi_1, \{Y_g\}, M_1)P(\Lambda|\xi_1, \{Y_g\}, M_1) = P(\Lambda, \hat{\Lambda}|\xi_1, \{Y_g\}, M_1)P(\hat{\Lambda}|\xi_1, \{Y_g\}, M_1).$$

By multiplying both sides by $P(\xi_1|\{Y_g\}, M_1)$ and integrating over θ_1,

$$\int P(\hat{\Lambda}, \Lambda|\xi_1, \{Y_g\}, M_1)P(\Lambda|\xi_1, \{Y_g\}, M_1)P(\xi_1|\{Y_g\}, M_1)d\theta_1$$
$$= \int P(\Lambda, \hat{\Lambda}|\xi_1, \{Y_g\}, M_1)P(\hat{\Lambda}|\xi_1, \{Y_g\}, M_1)P(\xi_1|\{Y_g\}, M_1)d\theta_1.$$

After rearrangement,

$$\int P(\hat{\Lambda}, \Lambda|\xi_1, \{Y_g\}, M_1)P(\Lambda, \xi_1|\{Y_g\}, M_1)d\theta_1$$
$$= P(\hat{\Lambda}|\{Y_g\}, M_1) \int P(\Lambda, \hat{\Lambda}|\xi_1, \{Y_g\}, M_1)P(\xi_1|\hat{\Lambda}, \{Y_g\}, M_1)d\theta_1.$$

It thus follows that

$$P(\hat{\Lambda}|\{Y_g\}, M_1) = \frac{E_1\{\alpha(\hat{\Lambda}, \Lambda|\xi_1, \{Y_g\}, M_1)q(\hat{\Lambda}|\Lambda)\}}{E_2\{\alpha(\Lambda, \hat{\Lambda}|\xi_1, \{Y_g\}, M_1)\}},$$

where the numerator expectation E_1 is with respect to the distribution $P(\Lambda, \xi_1|\{Y_g\}, M_1)$ and the denominator expectation E_2 is with respect to the distribution $P(\xi_1|\hat{\Lambda}, \{Y_g\}, M_1) \times q(\Lambda|\hat{\Lambda})$. This implies a simulation-consistent estimator

$$\widehat{P}(\hat{\Lambda}|\{Y_g\}, M_1) = \frac{\frac{1}{R}\sum_{r=1}^{R} \alpha(\hat{\Lambda}, \Lambda^{(r)}|\{Y_g\}, M_1)q(\hat{\Lambda}|\Lambda^{(r)})}{\frac{1}{J}\sum_{j=1}^{J} \alpha(\Lambda^{(j)}, \hat{\Lambda}|\xi_1^{(j)}, \{Y_g\}, M_1)}. \quad \text{(B.6)}$$

To estimate the numerator in Equation (B.6), we directly take model estimation MCMC sampling outputs $\{\Lambda^{(r)}\}$, $r = 1, \cdots, R$, where R denotes the length of MCMC. To estimate the denominator, we run additional J MCMC simulation to generate $\xi_1^{(j)}$ from the reduced conditional density $P(\xi_1|\hat{\Lambda}, \{Y_g\}, M_1)$ given the fixed value $\hat{\Lambda}$ and $\Lambda^{(j)}$ from $q(\Lambda|\hat{\Lambda})$. In practice, we set $J = R = 50,000$.

Calculating $P(\hat{\beta}|\hat{\Lambda}, \{Y_g\}, M_1)$ and $(\hat{\sigma}^2_\epsilon|\hat{\Lambda}, \hat{\beta}, \{Y_g\}, M_1)$ The second and third terms in Equation (B.5) are the reduced conditional densities. Since we know their full conditional density functions, we can estimate them by averaging the full conditional densities as follows,

$$\widehat{P}(\hat{\beta}|\hat{\Lambda}, \{Y_g\}, M_1) = \frac{1}{J}\sum_{j=1}^{J} P(\hat{\beta}|\hat{\Lambda}, \{Y_g\}, M_1, \sigma_\epsilon^{2(j)}, \{\tau_g^{(j)}\}), \tag{B.7}$$

$$\widehat{P}(\hat{\sigma}^2_\epsilon|\hat{\Lambda}, \hat{\beta}, \{Y_g\}, M_1) = \frac{1}{J}\sum_{j=1}^{J} P(\hat{\sigma}^2|\hat{\Lambda}, \hat{\beta}, \{Y_g\}, M_1, \{\tau_g^{(j)}\}), \tag{B.8}$$

where $\{\sigma_\epsilon^{2(j)}, \{\tau_g^{(j)}\}\}_{j=1}^J$ in Equation (B.7) are J draws that are obtained from a reduced Gibbs MCMC run in which Λ is fixed at $\hat{\Lambda}$ and the sampling is over $\{\beta, \sigma^2_\epsilon, \{\tau_g\}\}$, a procedure that requires no new programming. Similarly, $\{\{\tau_g^{(j)}\}\}_{j=1}^J$ are J draws from the reduced Gibbs MCMC run in which Λ and β are fixed at $\hat{\Lambda}$ and $\hat{\beta}$, respectively.

Calculating $P(\{\hat{\tau}_g\}|\hat{\Lambda}, \hat{\beta}, \hat{\sigma}^2_\epsilon, \{Y_g\}, M_1)$ The last term in Equation (B.5), $P(\{\hat{\tau}_g\}|\hat{\Lambda}, \hat{\beta}, \hat{\sigma}^2_\epsilon, \{Y_g\}, M_1)$, is a full conditional density, which is known.

We apply the above procedure to calculate the log marginal likelihoods of the altruistic social interactions model (M_1) and conventional social interactions model (M_2). Based on the estimation results for GPA in Table 3 of the paper, the log marginal likelihoods of Model M_1 (in Column (IV) of Table 3) and Model M_2 (in Column (I) of Table 3) are -3302.3 and -3311.4, respectively. Therefore, the corresponding Bayes factor between the two models is $\frac{P(\{Y_g\}|M_1)}{P(\{Y_g\}|M_2)} = \frac{\exp(-3302.3)}{\exp(-3311.4)} = 8955.3$. According to Kass and Raftery (1995), if M_2 is a nested model of M_1, then a Byes factor greater than 3 provides sufficient evidence that M_1 is preferred to M_2. Therefore, the altruistic social interactions model (M_1) performs better than the conventional social interactions model (M_2) according to Bayes factor. For the case of smoking, based on the estimation results in Table 4 of the paper, the log marginal likelihoods of model M_1 and M_2 are -10739.2 and -10750.6, respectively. The corresponding Bayes factor $\frac{P(\{Y_g\}|M_1)}{P(\{Y_g\}|M_2)} = 89321.7$ also confirms that the altruistic social interactions model fits the data better for the smoking outcome.

C Additional Results

Table C1: Monte Carlo Simulation Result – Heterogeneous Altruistic Social Interactions model with Endogenous networks

		Model (I)		Model (II)		Model (III)		Model (IV)	
Parameters	True	Bias	S.d.	Bias	S.d.	Bias	S.d.	Bias	S.d.
λ	0.0500	0.0060	0.0055	-0.0197	0.0130	0.0134	0.0069	0.0640	0.0076
ρ	0.5000	0.0249	0.1495	0.1219	0.0977	-	-	-	-
η	-0.3000	0.0108	0.0558	-	-	-	-	-	-
β_1	0.5000	0.0034	0.0165	0.0025	0.0160	0.0024	0.0174	0.0014	0.0172
β_2	0.2000	-0.0036	0.0130	0.0072	0.0137	-0.0073	0.0121	-0.0348	0.0140
δ_1	0.5000	0.1133	0.0229	0.0433	0.0284	0.0698	0.0232	-	-
δ_2	0.2000	-0.1239	0.0164	-0.0543	0.0265	-0.1116	0.0131	-	-
δ_3	0.5000	-0.0634	0.1449	0.3289	0.4159	-	-	-	-
δ_4	0.2000	0.0694	0.0693	0.0332	0.0871	-	-	-	-
γ_0	-2.2000	0.0112	0.0323	0.0251	0.0327	0.0869	0.0224	-	-
γ_1	0.3000	-0.0053	0.0206	-0.0030	0.0204	-0.0060	0.0226	-	-
γ_2	0.3000	0.0169	0.0458	0.0956	0.0635	-	-	-	-
γ_3	0.3000	0.0036	0.0453	-0.1634	0.1151	-	-	-	-
ζ	-1.0000	-0.0110	0.0422	-0.0393	0.0443	-0.0885	0.0327	-	-
σ^2	1.0000	-0.0380	0.0308	-0.1942	0.1125	0.1172	0.0250	0.5315	0.0308

Note: Model (I): Heterogeneous altruistic social interactions model, i.e., Equation (16) and Equation (19). Model (II): Heterogeneous altruistic social interactions model WITHOUT direct externality effect, i.e., Equation (16) and Equation (19) without η . Model (III): conventional social interactions model with endogenous networks. Model (IV): conventional social interactions model, i.e., Equation (3). We conduct a Monte Carlo simulation study with 100 repetitions. For each repetition, the point estimate is obtained from 20,000 MCMC draws with the first 2,000 draws dropped for the burn-in. The values shown for each parameter are the mean bias and the standard deviation from the point estimates across repetitions.

Table C2: Descriptive Statistics from The Original High School Sample

Variables	Min.	Max.	Mean	S.D.
GPA	1	4	2.8041	0.7900
Smoking	0	30	4.5210	9.9416
Male	0	1	0.4839	0.4997
Age	11	19	15.7672	1.2300
White	0	1	0.5860	0.4925
Black	0	1	0.1638	0.3701
Asian	0	1	0.0694	0.2541
Hispanic	0	1	0.1323	0.3388
Other race	0	1	0.0484	0.2146
Both parents	0	1	0.7450	0.4358
Less HS	0	1	0.0965	0.2953
HS	0	1	0.3067	0.4611
More HS	0	1	0.4455	0.4970
Edu missing	0	1	0.0814	0.2734
Professional	0	1	0.2782	0.4481
Other job	0	1	0.3664	0.4818
Welfare	0	1	0.0073	0.0853
Job missing	0	1	0.0724	0.2592
Homemaker	0	1	0.2057	0.4042
Group size	15	1829	563.4205	429.7314
Out-degree	0	10	3.4026	2.7885
In-degree	0	28	3.4026	3.2681
Network density	0.0000	0.2353	0.0187	0.0350
Number of groups (schools)		88		
Observations		49,590		

Note: Both parents means living with both parents. Less HS means student's mother has a lower than high-school degree. More HS means student's mother has a higher than high-school degree. The variables in italic are the reference categories in our estimation.

Table C3: Estimation Results for GPA: Altruistic Social Interactions Model with Endogenous Friendship Formation

	D1		D2		D3		D4	
λ	0.0449***		0.0432***		0.0371***		0.0381***	
	(0.0066)		(0.0069)		(0.0064)		(0.0082)	
λ^{I}	0.0299***		0.0289***		0.0251***		0.0270***	
	(0.0067)		(0.0065)		(0.0061)		(0.0075)	
η^{I}	-0.0696***		-0.0671***		-0.0557***		-0.0615***	
	(0.0207)		(0.0206)		(0.0190)		(0.0232)	
	Own	Contextual	Own	Contextual	Own	Contextual	Own	Contextual
Male	−0.1106***	0.0023	−0.1090***	0.0039	−0.1073***	0.0026	−0.1182***	0.0066
	(0.0275)	(0.0166)	(0.0273)	(0.0167)	(0.0273)	(0.0162)	(0.0274)	(0.0167)
Age	−0.0169	−0.0090***	−0.0200**	−0.0089***	−0.0205*	−0.0079***	−0.0185*	−0.0088
	(0.0117)	(0.0017)	(0.0112)	(0.0017)	(0.0121)	(0.0017)	(0.0117)	(0.0019)
Black	−0.0277	0.0046	−0.0397	0.0027	−0.0498	0.0051	−0.0163	−0.0005
	(0.0514)	(0.0141)	(0.0518)	(0.0146)	(0.0519)	(0.0146)	(0.0537)	(0.0152)
Asian	0.1422	0.0374	0.1355	0.0408	0.1245	0.0416	0.1379	0.0437
	(0.0960)	(0.0432)	(0.0948)	(0.0436)	(0.0946)	(0.0424)	(0.0934)	(0.0433)
Hispanic	−0.0466	0.0083	−0.0473	0.0100	−0.0436	0.0176	−0.0392	0.0150
	(0.0577)	(0.0281)	(0.0579)	(0.0284)	(0.0574)	(0.0285)	(0.0576)	(0.0280)
Other Race	−0.0319	−0.0011	−0.0340	−0.0100	−0.0266	−0.0004	−0.0218	0.0037
	(0.0581)	(0.0329)	(0.0584)	(0.0338)	(0.0573)	(0.0329)	(0.0574)	(0.0330)
Both parents	0.0900***	0.0354**	0.0870***	0.0349**	0.0843***	0.0370**	0.0849***	0.0308*
	(0.0305)	(0.0168)	(0.0301)	(0.0175)	(0.0304)	(0.0167)	(0.0307)	(0.0168)
Less HS	−0.0950**	−0.0649***	−0.0913**	−0.0581**	−0.0926**	−0.0613***	−0.0914**	−0.0581**
	(0.0425)	(0.0244)	(0.0444)	(0.0248)	(0.0431)	(0.0243)	(0.0431)	(0.0247)
More HS	0.1435***	0.0014	0.1483***	0.0115	0.1486***	0.0158	0.1457***	0.0082
	(0.0311)	(0.0149)	(0.0316)	(0.0155)	(0.0314)	(0.0153)	(0.0311)	(0.0150)
Edu missing	0.0216	−0.0278	0.0258	−0.0256	0.0153	−0.0309	0.0243	−0.0216
	(0.0531)	(0.0324)	(0.0530)	(0.0318)	(0.0526)	(0.0317)	(0.0535)	(0.0321)
Welfare	−0.0503	−0.1146	−0.0701	−0.1362	−0.0391	−0.0913	−0.0567	−0.0883
	(0.1229)	(0.0927)	(0.1232)	(0.0931)	(0.1227)	(0.0929)	(0.1221)	(0.0944)
Job missing	−0.0984*	−0.0083	−0.0957*	−0.0112	−0.0997*	−0.0108	−0.0994	−0.0142
	(0.0526)	(0.0306)	(0.0525)	(0.0304)	(0.0523)	(0.0299)	(0.0513)	(0.0298)
Professional	0.0313	−0.0271	0.0315	−0.0259	0.0319	−0.0245	0.0379	−0.0259
	(0.0364)	(0.0184)	(0.0365)	(0.0184)	(0.0359)	(0.0184)	(0.0364)	(0.0187)
Other job	−0.0156	0.0120	−0.0148	0.0146	−0.0190	0.0101	−0.0179	0.0076
	(0.0323)	(0.0168)	(0.0325)	(0.0170)	(0.0316)	(0.0168)	(0.0324)	(0.0172)
Z_1	0.0084	−0.0058	−0.0531	0.0032	−0.0312	−0.0021	0.0261	−0.0080
	(0.0176)	(0.0056)	(0.0203)	(0.0052)	(0.0303)	(0.0055)	(0.0194)	(0.0047)
Z_2			0.0415***	−0.0116**	0.0904***	−0.0115**	0.0537**	−0.0083
			(0.0198)	(0.0055)	(0.0297)	(0.0058)	(0.0270)	(0.0058)
Z_3					−0.0742***	−0.0087	0.1238***	−0.0203***
					(0.0280)	(0.0064)	(0.0203)	(0.0054)
Z_4							0.0667*	0.0005
							(0.0362)	(0.0075)
Network								
Age	0.9413***		0.7867***		0.6834***		0.7205***	
	(0.0235)		(0.0283)		(0.0307)		(0.0336)	
Sex	0.3392***		0.3379***		0.3324***		0.3188***	
	(0.0218)		(0.0238)		(0.0265)		(0.0288)	
Race	0.5760***		0.4005***		0.4700***		0.5273***	
	(0.0303)		(0.0361)		(0.0390)		(0.0432)	
$\|z_{i1} - z_{j1}\|$	-3.6327***		-3.1488***		-2.7547***		-2.6008***	
	(0.0817)		(0.0652)		(0.0649)		(0.0436)	
$\|z_{i2} - z_{j2}\|$			-2.8433***		-2.6319***		-2.5583***	
			(0.0649)		(0.0443)		(0.0408)	
$\|z_{i3} - z_{j3}\|$					-2.5459***		-2.5013***	
					(0.0549)		(0.0481)	

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Table – Continued

$\lvert z_{i4} - z_{j4} \rvert$				-1.9911***
				(0.0591)
σ_v^2	0.4522***	0.4497***	0.4389***	0.4361***
	(0.0119)	(0.0118)	(0.0127)	(0.0120)
AICM	88,734	83,398	81,522	81,739

Note: D_i, $i = 1, 2, 3, 4$ refers to the dimensions of the latent variables Z used in the network formation and outcome equations. The parameter estimates reported in this table are the posterior means and posterior standard deviations (in parentheses) computed on basis of 50,000 MCMC draws. We draw the first 5,000 draws for the burn-in. The asterisks ***(**,*) indicates that its 99% (95%, 90%) highest posterior density range does not cover zero.

Table C4: Estimation Results for smoking: Altruistic Social Interactions Model with Endogenous Friendship Formation

	D1		D2		D3		D4	
λ	0.0599***		0.0574***		0.0574***		0.0569***	
	(0.0102)		(0.0089)		(0.0093)		(0.0098)	
λ^{I}	0.0359***		0.0382***		0.0366***		0.0340***	
	(0.0100)		(0.0087)		(0.0090)		(0.0094)	
η^{I}	-0.2017***		-0.1904***		-0.1767***		-0.1764***	
	(0.0728)		(0.0668)		(0.0692)		(0.0682)	
	Own	Contextual	Own	Contextual	Own	Contextual	Own	Contextual
Male	−0.4138	−0.1841	−0.4419	−0.1947	−0.3846	−0.2347	−0.3425	−0.2013
	(0.3628)	(0.2157)	(0.3546)	(0.2151)	(0.3543)	(0.2133)	(0.3540)	(0.2129)
Age	0.8307***	−0.0498***	0.8626***	−0.0449**	0.8088***	−0.0420**	0.8482***	−0.0490***
	(0.1249)	(0.0165)	(0.1359)	(0.0170)	(0.1365)	(0.0165)	(0.1326)	(0.0163)
Black	−3.8043***	0.1848	−3.2300***	0.1349	−3.1854***	0.0142	−3.5953***	0.0141
	(0.6808)	(0.1900)	(0.6881)	(0.1949)	(0.6840)	(0.1980)	(0.6936)	(0.1933)
Asian	0.0359	−0.5421	0.2510	−0.4653	0.3822	−0.7343	0.0757	−0.5413
	(1.2321)	(0.5732)	(1.2403)	(0.5523)	(1.2190)	(0.5811)	(1.2300)	(0.5644)
Hispanic	−1.8004**	0.5954	−1.7115**	0.3352	−1.6029**	0.3883	−1.8562**	0.4037
	(0.7608)	(0.3656)	(0.7436)	(0.3720)	(0.7622)	(0.3731)	(0.7477)	(0.3646)
Other Race	0.5850	0.3255	0.7055	0.4043	0.6806	0.2382	0.6549	0.1979
	(0.7646)	(0.4366)	(0.7537)	(0.4313)	(0.7494)	(0.4367)	(0.7471)	(0.4377)
Both parents	−1.7755***	−0.2857	−1.7532***	−0.2406	−1.7092***	−0.2957	−1.7427***	−0.4110*
	(0.4089)	(0.2243)	(0.3982)	(0.2237)	(0.3977)	(0.2245)	(0.3989)	(0.2227)
Less HS	0.4994	0.2857	0.4632	0.2650	0.4986	0.2462	0.4222	0.3061
	(0.5794)	(0.3178)	(0.5696)	(0.3265)	(0.5697)	(0.3254)	(0.5736)	(0.3164)
More HS	−0.2177	0.3328*	−0.3001	0.1991	−0.2029	0.1879	−0.1949	0.3297
	(0.4138)	(0.1976)	(0.4088)	(0.1987)	(0.4084)	(0.2020)	(0.4089)	(0.2071)
Edu missing	0.1214	0.7360*	−0.0243	0.5811	0.0759	0.6576	−0.0569	0.8614**
	(0.6964)	(0.4261)	(0.6985)	(0.4223)	(0.7033)	(0.4161)	(0.6841)	(0.4293)
Welfare	2.0805	0.1576	1.9187	0.0146	2.0555	0.0777	1.6971	−0.0135
	(1.6329)	(1.2353)	(1.5921)	(1.2121)	(1.5936)	(1.2061)	(1.5979)	(1.1813)
Job missing	0.6286	0.6159	0.7265	0.6256	0.7284	0.7239*	0.7010	0.7161*
	(0.6898)	(0.4025)	(0.6883)	(0.4015)	(0.6836)	(0.3985)	(0.6775)	(0.3858)
Professional	0.5808	0.1414	0.5276	0.1570	0.6019	0.2308	0.5500	0.2352
	(0.4778)	(0.2369)	(0.4764)	(0.2377)	(0.4705)	(0.2389)	(0.4741)	(0.2376)
Other job	0.6403	0.0855	0.5905	0.0717	0.6323	0.1758	0.6401	0.2435
	(0.4244)	(0.2171)	(0.4163)	(0.2253)	(0.4204)	(0.2221)	(0.4086)	(0.2167)
Z_1	−0.6053***	0.0962	−1.4469***	0.1804***	−0.0828	0.0825	−2.4976***	0.2811***
	(0.2237)	(0.0671)	(0.2822)	(0.0709)	(0.3098)	(0.0744)	(0.2751)	(0.0670)
Z_2			0.7453***	−0.2760***	0.2004	0.0581	−1.0381***	0.2609***
			(0.2643)	(0.0717)	(0.4410)	(0.0923)	(0.2640)	(0.0689)
Z_3					−1.7947***	0.3508***	1.0764***	−0.1589**
					(0.4237)	(0.0812)	(0.3155)	(0.0739))
Z_4							−0.4839	0.0699
							(0.3422)	(0.0827)
Network								
Age	0.9150***		0.7449***		0.6967***		0.7520***	
	(0.0240)		(0.0286)		(0.0291)		(0.0364)	
Sex	0.3338***		0.3420***		0.3326***		0.3264***	
	(0.0220)		(0.0239)		(0.027)		(0.0285)	
Race	0.5484***		0.4290***		0.4653***		0.5325***	
	(0.0309)		(0.0340)		(0.041)		(0.0495)	
$\|z_{i1}-z_{j1}\|$	-3.6916***		-3.3265***		-2.7531***		-2.5928***	
	(0.0737)		(0.0820)		(0.0603)		(0.0534)	
$\|z_{i2}-z_{j2}\|$			-2.7755***		-2.6681***		-2.5166***	
			(0.0723)		(0.0544)		(0.0581)	
$\|z_{i3}-z_{j3}\|$					-2.4849***		-2.3229***	
					(0.0605)		(0.0454)	

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Table – Continued

$\lvert z_{i4} - z_{j4} \rvert$				-2.0793***
				(0.0737)
σ_v^2	78.4844***	76.7708***	76.3781***	73.1263***
	(2.1137)	(2.1259)	(2.1719)	(2.0931)
AICM	103,310	96,393	93,830	95,164

Note: D_i, $i = 1, 2, 3, 4$ refers to the dimensions of the latent variables Z used in the network formation and outcome equations. The parameter estimates reported in this table are the posterior means and posterior standard deviations (in parentheses) computed on basis of 50,000 MCMC draws. We draw the first 5,000 draws for the burn-in. The asterisks ***(**,*) indicates that its 99% (95%, 90%) highest posterior density range does not cover zero.

Table C5: Estimation Result: Directed Altruistic Social Interactions Models

	Directed Altruistic Model				Directed Altruistic Model w/ externality			
	GPA		Smoking		GPA		Smoking	
λ	0.0648^{***}		0.0765^{***}		0.0458^{***}		0.0564^{***}	
	(0.0071)		(0.0116)		(0.0075)		(0.0110)	
λ^{I}	0.0087^{***}		0.0025		0.0273^{***}		0.0306^{***}	
	(0.0021)		(0.0134)		(0.0093)		(0.0119)	
λ^{R}	-0.0044		0.0260^{*}		0.0012		0.0148	
	(0.0046)		(0.0144)		(0.0118)		(0.0138)	
η^{I}					-0.0563^{**}		-0.3007^{***}	
					(0.0287)		(0.0945)	
η^{R}					-0.0239		0.3792	
					(0.0388)		(0.2085)	
	Own	Contextual	Own	Contextual	Own	Contextual	Own	Contextual
Male	-0.1172^{***}	0.0036	−0.3119	−0.2366	-0.1150^{***}	0.0033	−0.3416	−0.1902
	(0.0278)	(0.0166)	(0.3563)	(0.2175)	(0.0270)	(0.0163)	(0.3586)	(0.2179)
Age	−0.0108	-0.0123^{***}	0.8603^{***}	-0.0591^{***}	−0.0146	-0.0089^{***}	0.8490^{***}	-0.0562^{***}
	(0.0113)	(0.0017)	(0.1317)	(0.0169)	(0.0109)	(0.0018)	(0.1338)	(0.0171)
Black	−0.0351	0.0059	-3.8138^{***}	0.2154	−0.0305	0.0024	-3.7602^{***}	0.1825
	(0.0523)	(0.0143)	(0.6701)	(0.1860)	(0.0525)	(0.0142)	(0.6794)	(0.1879)
Asian	0.1347	0.0404	−0.0134	−0.5625	0.1409	0.0389	0.0077	−0.5319
	(0.0936)	(0.0432)	(1.2483)	(0.5684)	(0.0930)	(0.0431)	(1.2416)	(0.5638)
Hispanic	−0.0487	0.0095	-1.7398^{**}	0.6309^{*}	−0.0452	0.0096	-1.7856^{**}	0.5564
	(0.0576)	(0.0275)	(0.7754)	(0.3718)	(0.0581)	(0.0280)	(0.7439)	(0.3657)
Other race	−0.0341	0.0020	0.6989	0.3409	−0.0341	−0.0015	0.6839	0.3454
	(0.0583)	(0.0331)	(0.7740)	(0.4323)	(0.0566)	(0.0330)	(0.7525)	(0.4382)
Both parents	0.0920^{***}	0.0343^{**}	-1.8452^{***}	−0.2218	0.0916^{***}	0.0366^{**}	-1.8053^{***}	−0.2794
	(0.0309)	(0.0170)	(0.4029)	(0.2231)	(0.0307)	(0.0174)	(0.3968)	(0.2199)
Less HS	-0.0964^{**}	-0.0647^{***}	0.5902	0.2203	-0.0937^{**}	-0.0641^{***}	0.4603	0.2624
	(0.0442)	(0.0250)	(0.5814)	(0.3210)	(0.0440)	(0.0240)	(0.5823)	(0.3188)
More HS	0.1459^{***}	−0.0012	−0.2242	0.3626	0.1443^{***}	0.0030	−0.2256	0.3496^{*}
	(0.0318)	(0.0151)	(0.4184)	(0.2010)	(0.0314)	(0.0150)	(0.4072)	(0.1994)
Edu missing	0.0218	−0.0295	0.0684	0.7035	0.0209	−0.0250	−0.0118	0.7032
	(0.0530)	(0.0326)	(0.7080)	(0.4369)	(0.0523)	(0.0314)	(0.6970)	(0.4300)
Welfare	−0.0522	−0.1244	2.1423	0.0432	−0.0518	−0.1143	2.1497	0.1281
	(0.1247)	(0.0942)	(1.6259)	(1.2217)	(0.1236)	(0.0931)	(1.5948)	(1.2103)
Job missing	-0.1021^{*}	−0.0098	0.7210	0.6641^{*}	-0.1000^{*}	−0.0085	0.6989	0.6676^{*}
	(0.0525)	(0.0301)	(0.7014)	(0.3997)	(0.0530)	(0.0305)	(0.6913)	(0.3973)
Professional	0.0268	−0.0305	0.6203	0.0916	0.0320	−0.0265	0.5920	0.1370
	(0.0373)	(0.0185)	(0.4811)	(0.2431)	(0.0366)	(0.0185)	(0.4818)	(0.2404)
Other job	−0.0189	0.0067	0.7144	0.0307	−0.0154	0.0123	0.6510	0.0717
	(0.0322)	(0.0170)	(0.4282)	(0.2190)	(0.0327)	(0.0166)	(0.4238)	(0.2164)
σ^2_ϵ	0.4558^{***}		79.2971^{***}		0.4525^{***}		78.5899^{***}	
	(0.0119)		(2.1347)		(0.0112)		(2.1244)	

Note: The parameter estimates reported in this table are the posterior means and posterior standard deviations (in parentheses) computed on the basis of 50,000 MCMC draws. We draw the first 5,000 draws for the burn-in. The asterisks ***(**,*) indicates that its 99% (95%, 90%) highest posterior density range does not cover zero.

Table C6: Estimation Results: Heterogeneous Altruistic Social Interactions Model with Endogenous Friendship Formation

Activity	GPA				Smoking			
	D3		D4		D3		D4	
λ	0.0492***		0.0470***		0.0747***		0.0739***	
	(0.0055)		(0.0067)		(0.0045)		(0.0048)	
ρ	0.6187***		0.6369***		0.4120*		0.3387	
	(0.2338)		(0.2278)		(0.2751)		(0.2602)	
η	-0.1303***		-0.1085***		-0.4344**		-0.4208**	
	(0.0221)		(0.0253)		(0.1842)		(0.1704)	
	Own	Contextual	Own	Contextual	Own	Contextual	Own	Contextual
Male	−0.1209***	0.0101	−0.1186***	0.0072	−0.1871	−0.1917	−0.0634	−0.2615
	(0.0271)	(0.0165)	(0.0266)	(0.0162)	(0.3551)	(0.2112)	(0.3448)	(0.2126)
Age	−0.0320***	−0.0097***	−0.0436***	−0.0084***	0.7580***	−0.0576**	0.7099***	−0.0492***
	(0.0118)	(0.0015)	(0.0136)	(0.0017)	(0.1319)	(0.0167)	(0.1359)	(0.0166)
Black	−0.0573	−0.0046	−0.0010	0.0060	−3.4775***	0.1567	−2.7953***	0.2163
	(0.0534)	(0.0146)	(0.0524)	(0.0144)	(0.6741)	(0.1833)	(0.6843)	(0.1834)
Asian	0.1030	−0.0008	0.0901	0.0051	−0.0532	−0.4186	0.0720	−0.5953
	(0.0949)	(0.0437)	(0.0927)	(0.0426)	(1.1285)	(0.5402)	(1.1479)	(0.5400)
Hispanic	−0.0604	0.0133	−0.0265	0.0257	−1.8048**	0.5007	−1.5848**	0.4833
	(0.0577)	(0.0283)	(0.0573)	(0.0285)	(0.7340)	(0.3602)	(0.7420)	(0.3647)
Other Race	−0.0399	−0.0106	−0.0227	0.0039	0.4595	0.3171	0.6785	0.4301
	(0.0586)	(0.0329)	(0.0567)	(0.0330)	(0.7206)	(0.4204)	(0.7158)	(0.4147)
Both parents	0.0944***	0.0388**	0.0842***	0.0359**	−1.8983***	−0.2393	−1.6849***	−0.2430
	(0.0313)	(0.0172)	(0.0301)	(0.0170)	(0.4043)	(0.2188)	(0.3928)	(0.2125)
Less HS	−0.0960**	−0.0534**	−0.0869**	−0.0459**	0.5740	0.1942	0.5574	0.2627
	(0.0441)	(0.0249)	(0.0433)	(0.0239)	(0.5690)	(0.3121)	(0.5659)	(0.3138)
More HS	0.1444***	0.0069	0.1352***	0.0151	−0.2244	0.3721*	−0.1183	0.3604*
	(0.0315)	(0.0149)	(0.0312)	(0.0147)	(0.4029)	(0.1929)	(0.3983)	(0.1926)
Edu missing	0.0282	−0.0165	0.0215	−0.0178	0.0724	0.6275	0.0274	0.6038
	(0.0525)	(0.0323)	(0.0522)	(0.0320)	(0.6706)	(0.4084)	(0.6608)	(0.4017)
Welfare	−0.0342	−0.1016	−0.0416	−0.0663	1.6425	0.0598	1.4548	0.3179
	(0.1234)	(0.0933)	(0.1221)	(0.0940)	(1.4234)	(1.1079)	(1.4249)	(1.1175)
Job missing	−0.0886	0.0076	−0.0902*	−0.0096	0.6909	0.7193*	0.7761	0.7044*
	(0.0521)	(0.0301)	(0.0508)	(0.0296)	(0.6727)	(0.3826)	(0.6769)	(0.3781)
Professional	0.0289	−0.0278	0.0303	−0.0377**	0.5876	0.1347	0.4279	0.0655
	(0.0360)	(0.0187)	(0.0362)	(0.0183)	(0.4675)	(0.2372)	(0.4569)	(0.2332)
Other job	−0.0169	0.0116	−0.0180	0.0137	0.7338	0.0745	0.6219	0.0029
	(0.0326)	(0.0167)	(0.0314)	(0.0167)	(0.4140)	(0.2109)	(0.4036)	(0.2133)
Z_1	−0.0987***	−0.0049	−0.1011***	0.0025	−2.5870***	0.4901***	−0.1807	0.0437
	(0.0309)	(0.0074)	(0.0224)	(0.0067)	(0.3474)	(0.0843)	(0.3145)	(0.0800)
Z_2	−0.0119	−0.0110	0.1032***	−0.0067	1.0381***	−0.2121**	0.7371	−0.1575
	(0.0245)	(0.0064)	(0.0244)	(0.0067)	(0.3493)	(0.0937)	(0.4570)	(0.1116)
Z_3	−0.0364	−0.0019	−0.0241	0.0001	0.1741	−0.0295	2.1789***	−0.2876**
	(0.0260)	(0.0068)	(0.0257)	(0.0070)	(0.3662)	(0.0888)	(0.3529)	(0.0931)
Z_4	−	−	−0.1451***	−0.0097	−	−	−2.6794***	0.4092***
			(0.0288)	(0.0081)			(0.3120)	(0.0914)
A	0.0155	−0.0144	0.0571	0.0029	2.4091***	0.0691	2.1153**	0.1226
	(0.0627)	(0.0188)	(0.0606)	(0.0187)	(0.9541)	(0.2273)	(0.9555)	(0.2324)
Network								
Age	0.6968***		0.7495***		0.6894***		0.7423***	
	(0.0321)		(0.0308)		(0.0355)		(0.0329)	
Sex	0.3561***		0.3616***		0.3646***		0.3549***	
	(0.0268)		(0.0291)		(0.0263)		(0.0329)	
Race	0.5344***		0.5449***		0.5011***		0.5203***	
	(0.0471)		(0.0479)		(0.0463)		(0.0576)	
$\lvert z_{i1} - z_{j1} \rvert$	-2.7467***		-2.6177***		-2.7889***		-2.6585***	
	(0.0421)		(0.0458)		(0.0496)		(0.0526)	

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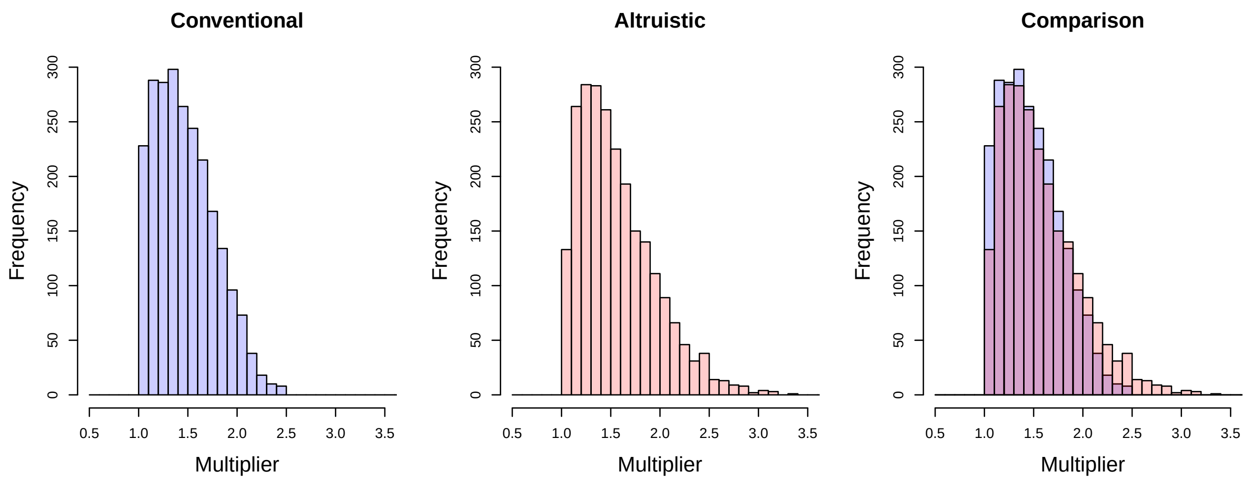
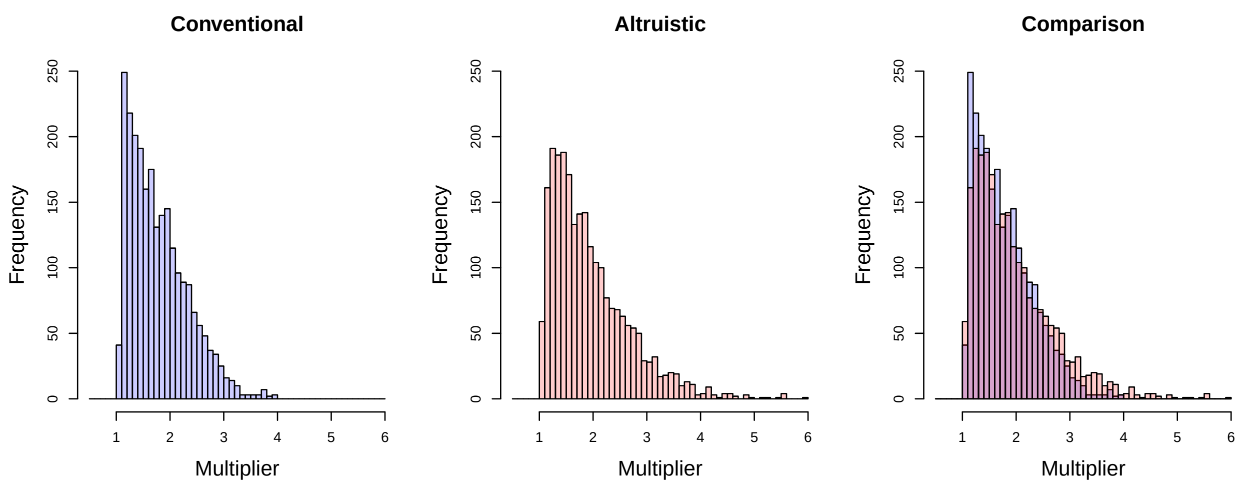
Table – Continued

$\lvert z_{i2} - z_{j2} \rvert$	-2.7122***	-2.5284***	-2.7241***	-2.5789***
	(0.0404)	(0.0428)	(0.0414)	(0.0532)
$\lvert z_{i3} - z_{j3} \rvert$	-2.6476***	-2.4735***	-2.6471***	-2.4929***
	(0.0446)	(0.0446)	(0.0494)	(0.0545)
$\lvert z_{i4} - z_{j4} \rvert$	-	-2.4260***	-	-2.3343***
		(0.0460)		(0.0675)
a_{ig}	0.1412***	0.1753***	0.1357***	0.1341***
	(0.0424)	(0.0494)	(0.0447)	(0.0532)
a_{jg}	1.4109***	1.4019***	1.3834***	1.4246***
	(0.0448)	(0.0498)	(0.0436)	(0.0541)
σ_v^2	0.4446***	0.4175***	72.6769***	69.7159***
	(0.0125)	(0.0121)	(2.2695)	(2.2621)
AICM	80,505	83,335	89,472	93,697

Note: D_i, $i = 3, 4$ refers to the dimensions of the latent variables Z used in the network formation and outcome equations. The parameter estimates reported in this table are the posterior means and posterior standard deviations (in parentheses) computed on basis of 50,000 MCMC draws. We draw the first 5,000 draws for the burn-in. The asterisks ***(**,*) indicates that its 99% (95%, 90%) highest posterior density range does not cover zero.

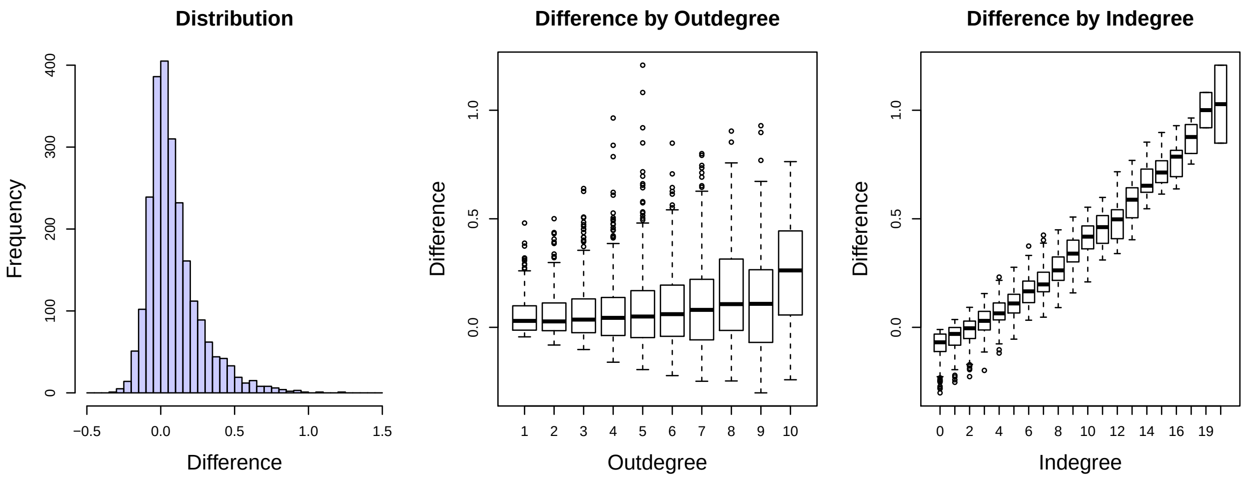


(a) Histogram of multiplier effects with regard to GPA

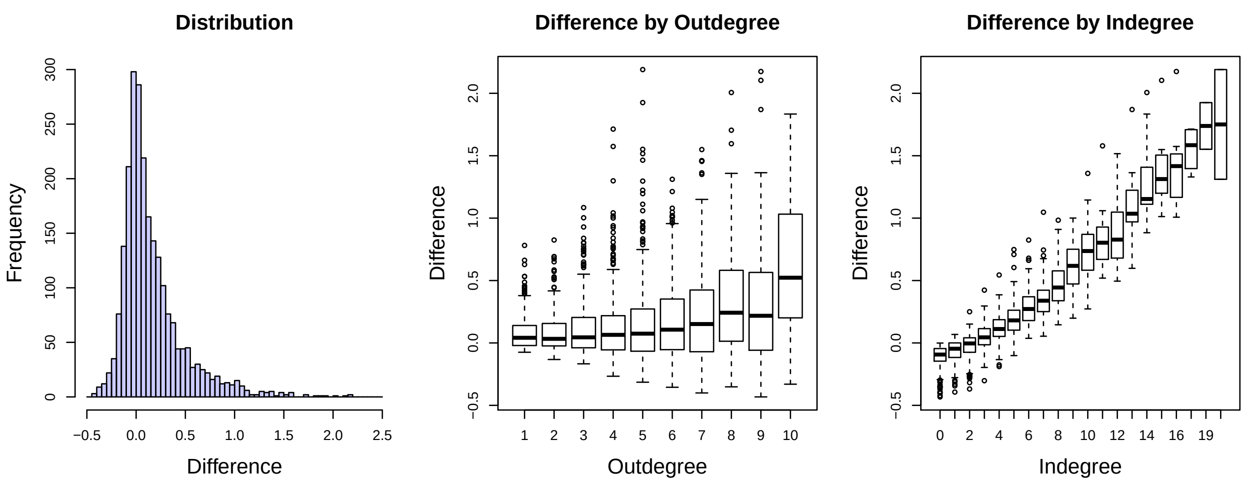


(b) Histogram of multiplier effects with regard to smoke

Figure C1: Histogram of Multipler Effects



(a) Difference of multiplier effects with regard to GPA



(b) Difference of multiplier effects with regard to smoke

Figure C2: Difference of Multipler Effects

References

Casella, G. (1985). An introduction to empirical bayes data analysis. *The American Statistician*, 39(2):83–87.

Chib, S. (1995). Marginal likelihood from the gibbs output. *Journal of the American Statistical Association*, 90(432):1313–1321.

Chib, S. and Jeliazkov, I. (2001). Marginal likelihood from the metropolis–hastings output. *Journal of the American Statistical Association*, 96(453):270–281.

Kass, R. E. and Raftery, A. E. (1995). Bayes factors. *Journal of the American Statistical Association*, 90(430):773–795.