Online Appendix for “Estimating the U.S. Output Gap with State-Level Data”

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[1] The views expressed in this paper are solely the responsibility of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System.

Online Appendix

A Contacts with the Literature

This work is related to the literature on trend-cycle decompositions using UC models. It also relates to the literature on common components estimation with dynamic factor models.

Introducing multidimensional variability into the UC model's setup can be done in three ways. One is to incorporate many variables belonging to one unit of interest to extract common trends and/or cycles. Another possibility is to incorporate many units of interest that use the same variable to extract common trends, cycles, or both. The third possibility considers a combination of many units of interest and many variables to extract common trends, cycles, or both. In this paper, I adopt the third possibility.

A.1 Many Variables and a Single Economic Unit

Crone and Clayton-Matthews (2005) employ the UC model discussed in Stock and Watson (1988, 1989) to describe how to use mixed frequency data from the U.S. states to estimate state-level monthly indexes of economic activity for each state. Observable variables for each state are the first difference of the following: (the log of) nonagricultural employment, the unemployment rate, (the log of) average hours worked in manufacturing, and (the log of) real wage and salary disbursements. A scalar latent stationary series is common to the state-level observable variables and is interpreted as the state's cycle. The observable series load on the state's cycle with leads and/or lags. In this analysis, only the cross-sectional structure of the many variables is exploited, not the variability along the state-level dimension.

Basistha and Startz (2008) and Fleischman and Roberts (2011) also use several variables, but at the aggregate level, to estimate the U.S. NAIRU and the U.S. potential output and its associated business cycle, respectively. The authors of both papers emphasize the advantages of using a multivariate approach in the estimation of UC models, including better estimate precision and the ability to coherently assess the trade-offs of competing signals.

A.2 A Single Variable and Many Economic Units

Kouparitsas (2002, 2001) specifies a UC model to decompose U.S. regional per capita income fluctuations into their trend and cycle components. Using data from the eight BEA regions, the model assumes that real income in each region is the sum of region-specific trend and cyclical components. The trend is assumed to follow a unit root with drift, whereas the cyclical component is made up of a common cycle across regions as well as a regional cycle. Each region has a different sensitivity with respect to the common cycle. The estimation of this UC model provides an estimated U.S. business cycle that accords well with National Bureau of Economic Research (NBER) recession periods and region-specific counterparts. Overall, the results suggest that a large share of the regional business cycle variations is explained by the common component and that spillovers from one region to another are not a significant source of variations. Even though Kouparitsas assumes a common cycle, he does not assume the existence of a common trend. This assumption makes his model less general than the model I propose, which also incorporates the unemployment rate in addition to data on real GDP to inform the estimation of the cycle and allows one to interpret the estimated cycle as a measure of the output gap in contrast with the cycle obtained from real income data.

Del Negro and Otrok (2008) extend a factor model to incorporate time-varying factor loadings and stochastic volatility to extract the international business cycle using a panel of 19 countries. The model is estimated with Bayesian methods and allows one to obtain the common and country-specific cycles for the GDP growth rates, but it does not consider the common GDP trends.

Mitra and Sinclair (2012) also use many units and one variable. They propose a multivariate UC model to simultaneously decompose real GDP for each of the G-7 countries into its respective trend and cycle components. The setup considers real GDP as the only observable variable and assumes that each country's GDP is driven by specific trend and cycle components. The setup allows for possible correlations between any of the contemporaneous shocks to the unobserved (trend and cycle) components.

Stock and Watson (2016) propose a multivariate dynamic factor model with time-varying coefficients and stochastic volatility estimated with Bayesian methods to calculate the U.S. trend inflation. They use 17 components of the personal consumption expenditure price inflation to construct an index akin to core inflation. This work is the most similar to the present paper, although Stock and Watson's paper only considers one variable in the analysis—the inflation rate—while this paper considers two—real GDP and the unemployment rate, which are linked in a structural way assuming Okun's law to provide more information for estimating the cyclical component of aggregate output. Moreover, the framework of Stock and Watson is not suitable to analyze business cycle fluctuations because the common and idiosyncratic shocks are all assumed to not have serial correlation.

A.3 Many Variables and Many Economic Units

Gregory, Head and Raynauld (1997) use a dynamic factor model estimated with classical methods to decompose aggregate output, consumption, and investment for the G-7 countries into factors that are (i) common across all countries and aggregates, (ii) common across aggregates within a country, and (iii) specific to each individual aggregate. The authors have to detrend the data to use the dynamic factor models approach because the underlying factors are assumed to be stationary. Similarly, Kose, Otrok and Whiteman (2003) estimate a dynamic factor model, but with Bayesian methods, to extract common components from macroeconomic aggregates (output, consumption, and investment) in a 60-country sample covering seven regions of the world. They allow factors common to the world, the regions, and the countries. Here, too, data are de-trended.

For the U.S., Owyang, Rapach and Wall (2009) use state-level income and payroll employment data to estimate a dynamic factor model of the 48 contiguous states and the District of Columbia in order to extract business cycle factors. The estimation of the model identifies three common factors underlying the fluctuations in state-level income and employment growth, with the first common factor resembling aggregate fluctuations in real activity at the national level. The factors explain a large proportion of the total variability in state-level variables, although there is still a substantial amount of cross-state heterogeneity.

B State-space Model in Matrix Form

The model in equation (1)-equation (11) can be written in matrix form as the following:

$$\mathbf{z}_{it} = \mathrm{C}(\mathbf{\Theta}_{\mathrm{mi}}) + \mathrm{H}(\mathbf{\Theta}_{\mathrm{mi}})\mathbf{x}_{it} + \mathbf{w}_{it}, \qquad \mathbf{w}_{it}|\mathfrak{F}_{t-1} \sim \text{ iid } \mathbb{N}(0, \mathrm{R}(\mathbf{\Theta}_{\mathrm{mi}})) \tag{B.1}$$

$$\mathbf{x}_{it} = \mathrm{F}(\mathbf{\Theta}_{\mathrm{si}})\mathbf{x}_{i,t-1} + \mathbf{G}\mathbf{v}_{it}, \qquad \mathbf{v}_{it}|\mathfrak{F}_{t-1} \sim \text{ iid } \mathbb{N}(0, \mathrm{Q}(\mathbf{\Theta}_{\mathrm{si}})), \tag{B.2}$$

where

$$\mathbf{z}_{it} = \begin{bmatrix} \Delta y_{it} \\ \Delta u_{it} \end{bmatrix}, \qquad \mathbf{x}_{it} = \begin{bmatrix} c_t \\ c_{t-1} \\ c_{t-2} \\ \eta_t^y \\ \eta_t^u \\ \upsilon_{it} \\ \upsilon_{i,t-1} \\ \upsilon_{i,t-2} \end{bmatrix}, \qquad \mathbf{w}_{it} = \begin{bmatrix} \eta_{it}^y \\ \eta_{it}^u \end{bmatrix}, \qquad \mathbf{v}_{it} = \begin{bmatrix} \varepsilon_t \\ \eta_t^y \\ \eta_t^u \\ \zeta_{it} \end{bmatrix},$$

$$\mathrm{C}(\mathbf{\Theta}_{\mathrm{mi}}) = \begin{bmatrix} \mu_i + \delta_i^y \mu \\ 0 \end{bmatrix},$$

$$\mathrm{H}(\mathbf{\Theta}_{\mathrm{mi}}) = \begin{bmatrix} \alpha_i & -\alpha_i & 0 & \delta_i^y & 0 & 1 & -1 & 0 \\ \alpha_i\theta_{1i} & \alpha_i(\theta_{2i} - \theta_{1i}) & -\alpha_i\theta_{2i} & 0 & \delta_i^u & \theta_{1i} & \theta_{2i} - \theta_{1i} & -\theta_{2i} \end{bmatrix},$$

$$\mathrm{R}(\mathbf{\Theta}_{\mathrm{vi}})) = \begin{bmatrix} \sigma^2_{\eta_i^y} & 0 \\ 0 & \sigma^2_{\eta_i^u} \end{bmatrix},$$

$$
\mathrm{F}(\mathbf{\Theta}_{\mathrm{si}}) = \begin{bmatrix}
\phi_1 & \phi_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \rho_i & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix},
$$

$$
\mathbf{G} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix},
$$

$$
\mathrm{Q}(\mathbf{\Theta}_{\mathrm{vi}}) = \begin{bmatrix}
\sigma_\varepsilon^2 & 0 & 0 & 0 \\
0 & \sigma_{\eta^y}^2 & 0 & 0 \\
0 & 0 & \sigma_{\eta^u}^2 & 0 \\
0 & 0 & 0 & \sigma_{\zeta_i}^2
\end{bmatrix},
$$

and

$$
\mathbf{\Theta}_{\mathrm{mi}} = \{\mu, \mu_i, \alpha_i, \theta_{1i}, \theta_{2i}, \delta_i^y, \delta_i^u, \sigma^2_{\eta_i^y}, \sigma^2_{\eta_i^u}\}
$$
$$
\mathbf{\Theta}_{\mathrm{si}} = \{\phi_1, \phi_2, \rho_i, \sigma_e^2, \sigma^2_{\eta^y}, \sigma^2_{\eta^u}, \sigma^2_{\zeta_i}\},
$$

for $i = 1, \ldots, n$.

C Details on the Gibbs Sampler

Let $\mathbf{z}_{it}$, $\mathbf{x}_{it}$, $\mathbf{\Theta}_{\text{mi}}$, and $\mathbf{\Theta}_{\text{si}}$ for $i = 1, 2, \ldots, n$, be defined as in Appendix B. Let $\mathbf{Z}_T = \{\tilde{\mathbf{z}}_1, \tilde{\mathbf{z}}_2, \ldots, \tilde{\mathbf{z}}_T\}$ denote the observed data and let $\mathbf{X}_T = \{\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, \ldots, \tilde{\mathbf{x}}_T\}$. Here, $\tilde{\mathbf{z}}_t = \{\mathbf{z}_{1t}, \mathbf{z}_{2t}, \ldots, \mathbf{z}_{nt}\}$ and $\tilde{\mathbf{x}}_t = \{\mathbf{x}_{1t}, \mathbf{x}_{2t}, \ldots, \mathbf{x}_{nt}\}$. Denote $\mathbf{\Theta}_{\text{m}} = \bigcup_{\text{i}=1}^{n} \mathbf{\Theta}_{\text{mi}}$ and $\mathbf{\Theta}_{\text{s}} = \bigcup_{\text{i}=1}^{n} \mathbf{\Theta}_{\text{si}}$.

Partition $\mathbf{\Theta}_{\text{si}} = \mathbf{\Theta}_{\text{si}}^1 \bigcup \mathbf{\Theta}_{\text{si}}^2 \bigcup \mathbf{\Theta}_{\text{si}}^3$, where

$$
\begin{aligned}
\mathbf{\Theta}_{\text{si}}^1 &= \{\phi_1, \phi_2\}, \\
\mathbf{\Theta}_{\text{si}}^2 &= \{\rho_i, \sigma_{\zeta_i}^2\}, \\
\mathbf{\Theta}_{\text{si}}^3 &= \{\sigma_e^2, \sigma_{\eta^y}^2, \sigma_{\eta^u}^2\}.
\end{aligned}
$$

Notice that the identification conditions imply that $\mathbf{\Theta}_{\text{si}}^3$ is not random.

Also, partition $\mathbf{\Theta}_{\text{mi}} = \mathbf{\Theta}_{\text{mi}}^1 \bigcup \mathbf{\Theta}_{\text{mi}}^2$, where

$$
\begin{aligned}
\mathbf{\Theta}_{\text{mi}}^1 &= \{\mu_i, \alpha_i, \theta_{1i}, \theta_{2i}, \delta_i^y, \delta_i^u\}, \\
\mathbf{\Theta}_{\text{mi}}^2 &= \{\sigma_{\eta_i^y}^2, \sigma_{\eta_i^u}^2\},
\end{aligned}
$$

where μ has been excluded because it is fixed under the identification conditions.

The Gibbs sampler procedure is as follows:

1. Start with initial values for the model's parameters, $\mathbf{\Theta} = \mathbf{\Theta}_{\text{m}} \bigcup \mathbf{\Theta}_{\text{s}}$.

2. Draw $\mathbf{X}_T$ from $p(\mathbf{X}_T | \mathbf{Z}_T, \mathbf{\Theta}_{\text{m}}, \mathbf{\Theta}_{\text{s}})$ using the Durbin and Koopman (2002) simulation smoother.

3. Draw $\mathbf{\Theta}_{\text{s}}^1$ from $p(\mathbf{\Theta}_{\text{s}}^1 | \mathbf{Z}_T, \mathbf{X}_T, \mathbf{\Theta}_{\text{m}}, \mathbf{\Theta}_{\text{si}}^2, \mathbf{\Theta}_{\text{si}}^3)$ using the conditional distributions implied by the independent normal-inverse-gamma prior.

4. For $i = 1, 2, \ldots, n$, sample as follows:

   (a) Draw $\mathbf{\Theta}_{\text{si}}^2$ from $p(\mathbf{\Theta}_{\text{si}}^2 | \mathbf{Z}_T, \mathbf{X}_T, \mathbf{\Theta}_{\text{m}}, \mathbf{\Theta}_{\text{s}}^1, \mathbf{\Theta}_{\text{si}}^2)$ using the conditional distributions implied by the independent normal-inverse-gamma prior.

   (b) Draw $\mathbf{\Theta}_{\text{mi}}^1$ from $p(\mathbf{\Theta}_{\text{mi}}^1 | \mathbf{Z}_T, \mathbf{X}_T, \mathbf{\Theta}_{\text{mi}}^2, \mathbf{\Theta}_{\text{mi}}^3, \mathbf{\Theta}_{\text{m}(/\text{i})}, \mathbf{\Theta}_{\text{s}})$ using the conditional distributions implied by the independent normal-inverse-gamma prior. Repeat similarly for $\mathbf{\Theta}_{\text{mi}}^2$ and sample from

- $p(\mathbf{\Theta}^2_{\text{mi}}|\mathbf{Z}_T, \mathbf{X}_T, \mathbf{\Theta}^1_{\text{mi}}, \mathbf{\Theta}^3_{\text{mi}}, \mathbf{\Theta}_{\text{m}(/\text{i})}, \mathbf{\Theta}_{\text{s}})$.

5. Return to step 2.

D States Most Strongly Affected by the Great Recession and Growth Rates by States after the Great Recession

The states that were most strongly affected by the Great Recession in terms of real GDP growth rates in the 2007–09 period appear in figure M.1. Arizona, Florida, Michigan, and Nevada experienced the lowest average annualized growth rates of real GDP, with rates lower than negative 3 percent. By contrast, states such as Alaska, South Dakota, and North Dakota experienced average growth rates greater than 3 percent in that period (not shown). All told, 26 of the 50 states plus the District of Columbia experienced negative average growth rates. Figure M.2 shows the average output growth rates that the states experienced during and after the recovery between 2010 and 2016. North Dakota and Texas have grown the fastest, with average rates above 3 percent, while Wyoming and Alaska have grown the slowest, with average rates close to negative 1 percent.

[Figure 1 about here.]

Several different factors can help to rationalize the heterogeneity in the growth rates of the U.S. states during and after recessions. Industry composition, demographics, credit demand and supply, and state fiscal policies, among others have been mentioned in the literature (see Owyang, Piger and Wall, 2005; Carlino and Defina, 1998; Mian and Sufi, 2009, 2010; Owyang and Zubairy, 2013, for example). I exploit the heterogeneity in the data at the state level to obtain a common estimate of the cycle—the aggregate output gap—while at the same time I relate the aforementioned factors to characterize the output gaps at the state level in the forthcoming sections.

E Prior Distributions

The prior distributions of the parameters of the UC model with state-level data appear in table M.1. The equations for each parameter are in the last column.

[Table 1 about here.]

The prior means of the parameters of the common cycle, ϕ_1 and ϕ_2, are similar to those found in the literature on trend-cycle decompositions of output. For example, Morley, Nelson and Zivot (2003) find that the estimation of a UC model with aggregate data on real GDP in absence of correlation between trend and cycle innovations yields estimated coefficients $\hat{\phi}_1 = 1.53$ and $\hat{\phi}_2 = -0.61$. Similarly, Gonzalez-Astudillo and Roberts (2016) also use aggregate data but include the unemployment rate along with real GDP and find estimates around 1.6 for ϕ_1 and -0.65 for ϕ_2 both with or without correlation between output innovations. In the present paper, the joint prior distribution of these two parameters is truncated to satisfy the weak stationarity feature of the common cycle. The mean growth rate of each state's GDP has a truncated normal prior distribution with mean 0.8, which yields an annualized growth rate of real GDP around 3 percent, roughly the historical average. The distributions of the parameters that load on the common cycle, α_i, are normal with means and standard deviations equal to one.[1] The coefficients that load on the common trends, δ_i^y, and δ_i^u, have truncated normal distributions with means and standard deviations equal to one. The Okun's law coefficients, θ_{1i} and θ_{2i}, are normally distributed with prior means equal to -0.25 each, such that the long-run Okun's law coefficient for each state is centered at -0.5 under the prior distribution. This is the usual Okun's law coefficient used in the literature (see Abel, Bernanke and Croushore, 2013). The prior distribution of the parameter of the idiosyncratic cycle, ρ_i, is a truncated standard normal. Finally, given the lack of previous estimates of these coefficients in the literature, I assume that $\sigma^2_{\eta_i^y}$, $\sigma^2_{\eta_i^u}$, and $\sigma^2_{\zeta_i}$ are distributed as inverse-gamma centered at one with undefined variance. In general, the standard deviations of the parameters imply that the distributions are neither too tight nor too narrow.[2]

F Parameter Results and Convergence Diagnostics

This appendix lays out the results from the Bayesian estimation showing the estimates of the posterior mean and standard deviation of the parameters of the model, as well as the first and fiftieth order autocorrelation coefficient, the relative numerical efficiency (RNE) using a 4 percent taper, and the p-value of the Geweke (1991) convergence diagnostics using a 4 percent tapper as

[1] In the case of California, the prior distribution of α_i is a truncated normal with mean and standard deviation equal to one.

[2] I also explored assuming flat priors for $\sigma^2_{\eta_i^y}$, $\sigma^2_{\eta_i^u}$, and $\sigma^2_{\zeta_i}$. The results do not change materially.

well in which the null hypothesis considers equality of the means of the first 20 percent of draws with that of the last 50 percent. Given the large number of parameters, the results appear in tables M.2-M.11. In the tables, the parameters μ_i, α_i, δ_i^y, δ_i^u, θ_{1i}, θ_{2i}, ρ_i, $\sigma^2_{\zeta_i}$, $\sigma^2_{\eta_i^y}$, and $\sigma^2_{\eta_i^u}$ are numbered from $i = 1, \ldots, 51$ according to the states and the District of Columbia in the following order: Alabama, Alaska, Arizona, Arkansas, California, Colorado, Connecticut, Delaware, District of Columbia, Florida, Georgia, Hawaii, Idaho, Illinois, Indiana, Iowa, Kansas, Kentucky, Louisiana, Maine, Maryland, Massachusetts, Michigan, Minnesota, Mississippi, Missouri, Montana, Nebraska, Nevada, New Hampshire, New Jersey, New Mexico, New York, North Carolina, North Dakota, Ohio, Oklahoma, Oregon, Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, Texas, Utah, Vermont, Virginia, Washington, West Virginia, Wisconsin, and Wyoming. ϕ_1 and ϕ_2 are the parameters of the AR(2) specification of the common component of the cycle.

The draws from the posterior distribution used to produce the results of the model are based on 300,000 draws after burning in the first 100,000 and thinning every 100th draw, which left me with 2,000 draws from the posterior distribution. The results of the diagnostics tests show that these 2,000 draws do not evidence significant autocorrelation of first order and almost no autocorrelation of order fifty. Apart from 3 out of 513 parameters, the p-values of the test of equality of means between the fist 20 percent of the draws and the last 50 percent are all above 1 percent, which indicate that the null hypothesis is not rejected for any of the parameters and the sampler has converged.

[Table 2 about here.]

[Table 3 about here.]

[Table 4 about here.]

[Table 5 about here.]

[Table 6 about here.]

[Table 7 about here.]

[Table 8 about here.]

[Table 9 about here.]

[Table 10 about here.]

[Table 11 about here.]

G Obtaining the Aggregate Trend and Cycle

The objective of the estimation is to obtain the trend-cycle decomposition of aggregate GDP by exploiting the cross-sectional variability of state-level data. Because of the nonlinearity implicit in the aggregation of the variables and the fact that state-level GDP appears in logs in the specification equation (1)-equation (11), an approximation is needed. The quarterly growth rate of aggregate real GDP is given by the following:

$$\begin{aligned}
\Delta\%Y_t &= \sum_{i=1}^{n} w_{it}\Delta\%Y_{it} \\
&\approx \sum_{i=1}^{n} w_{it}\Delta y_{it} \\
&= \sum_{i=1}^{n} w_{it}\left(\Delta\tau_{it}^{y} + \Delta c_{it}\right) \\
&= \sum_{i=1}^{n} w_{it}\left(\delta_i^y(\mu + \eta_t^y) + \mu_i + \eta_{it}^y + \alpha_i\Delta c_t + \Delta\upsilon_{it}\right) \\
&\approx \underbrace{\bar{\delta^y}(\mu + \eta_t^y) + \bar{\mu} + \bar{\eta^y}_t}_{\Delta\text{ GDP Trend}} + \underbrace{\bar{\alpha}\Delta c_t + \bar{\upsilon}_t^y}_{\Delta\text{ GDP Cycle}} \quad ,
\end{aligned}$$

where Y_{it} is real GDP of state i, Y_t is aggregate real GDP in period t, and the contribution of of state's i GDP to aggregate GDP is denoted by w_{it}.

Hence, I can express the trend and cycle components of the aggregate GDP as

$$\text{GDP Trend} \approx \bar{\delta^y}\tau_t^y + \bar{\xi}_t^y,$$

$$\text{GDP Cycle} \approx \bar{\alpha}c_t + \bar{\upsilon}_t^y,$$

where

$$\bar{\mu}_t = \sum_{i=1}^{n} w_i \mu_i,$$
$$\bar{\delta^y} = \sum_{i=1}^{n} w_i \delta_i^y,$$
$$\bar{\eta^y}_t = \sum_{i=1}^{n} w_i \eta_{it}^y,$$
$$\bar{\xi}_t^y = \sum_{i=1}^{n} w_i \xi_{it}^y,$$
$$\bar{\alpha} = \sum_{i=1}^{n} w_i \alpha_i,$$
$$\bar{\upsilon}_t^y = \sum_{i=1}^{n} w_i \upsilon_{it},$$

where $w_i = \sum_{t=1}^{T} w_{it}/T$ is the sample average of each state's contributions to aggregate GDP. I use the smoothed estimates $c_{t|T}$ and $\upsilon_{it|T}$ to obtain the estimate of the cycle and, by residual, the trend.

H Variance Decomposition of States' Cycles and Trends

In this section, I describe how each state's output cycle and trend variability are explained by the variability in the common cycle and trend during the period of analysis. To that end, I perform variance decompositions of the state's cycle and trend of output. Recall that the specification for the cycle of each state $i = 1, 2, \ldots, n$ is $c_{it} = \alpha_i c_t + \upsilon_{it}$. Hence, one can obtain the fraction of the variance of the cycle that is due to the common cycle and the fraction that corresponds to the idiosyncratic component. Similarly, because the output trend of each state is given by $\tau_{it}^y = \delta_i^y \tau_t^y + \xi_{it}^y$, one can obtain the proportion of the variability of the output trend of each state that is due to the common trend and that due to the idiosyncratic component. Figures M.3 and M.4 show the percent of the variance decomposition of the state's cycles and output trends, respectively.

[Figure 2 about here.]

Economic policies designed at the federal level can have different effects on the state economies, depending, among several factors, on industry composition, size of firms, the ability of banks to alter their balance sheets, demographics, and government spending composition as described by Carlino and DeFina (1999) and Owyang and Zubairy (2013). In that regard, the propagation of federal economic policies at the state level can be different depending, for instance, on how strongly a particular state's cycle or trend is linked to its common counterpart, which I assumed is the target of economic policies at the federal level. On the one hand, Nevada, California, Georgia, Arizona, and Florida are among the states with cycle variability that is explained the most by the variance of the common cycle, whereas West Virginia, Wyoming, Louisiana, and North Dakota have the smallest variation of their cycle attributed to the common cycle. On the other hand, the states whose trends are most strongly connected with the common trend are Kentucky, Alabama, and Indiana, whereas those with lowest associations are New York, Delaware, and Alaska.

I Variance Decomposition of States' GDP Growth

The model also allows one to establish a measure of how cyclical the state economies are. In this section, I decompose the variance of real GDP growth at the state level in the proportion that is explained by variations of the cycle and the proportion that is explained by variations of the output trend during the period of analysis. This indicator is useful to characterize the sources of variations in GDP growth for policymaking decisions both at the state and the federal level, for example. Figure M.5 illustrates the proportions for each state.

[Figure 3 about here.]

California, Utah, Florida, Arizona, Georgia, and Oregon have the highest proportion of their real GDP growth's variance explained by the variance of their respective output cycles. In contrast, states such as North Dakota, Wyoming, Vermont, and West Virginia have the lowest proportions. One can also characterize the features that make a state "more cyclical" than others. Table M.13 shows that, on average, and at the 10 percent level of significance, states with higher participation of the sectors related with wholesale trade, retail trade, and transportation are more cyclical, likewise states that have a higher share of population between the ages of 18 and 44. In addition, states with higher leverage tend to experience fluctuations in output that are explained more by fluctuations in their cyclical component than in their trend output.

J Okun's Law Coefficients at the State Level

As a final point, the results shed light on the cyclical features of the state-level labor markets, in particular the sensitivity of the unemployment gap to the output gap in each state. Policies designed to affect the cyclical position of the economy at the federal level can propagate differently across states' labor markets via two channels. First, these policies can affect the state output gaps differently. Second, the state labor markets can have unique reactions to their respective output gaps. I describe the first channel in online Appendix H. I measure the second channel by the sum of the Okun's law coefficients, $\theta_{1i} + \theta_{1i}$. The higher the absolute value of this sum, the more responsive the unemployment rate is to the cyclical fluctuations of output at the state level. Figure M.6 presents the results grouped by values of the posterior mean estimates.

[Figure 4 about here.]

According to the estimates of the model, the states with more-cyclical labor markets are Louisiana, Rhode Island, and Mississippi whereas the states with less-cyclical labor markets are Nebraska, Kansas, and Alaska. Guisinger et al. (2018) find that Lousiana has the highest comparable Okun's law coefficient, just as in this paper. Rhode Island also appears in the top three states classified by their Okun's law coefficients. On the other extreme, Guisinger et al. find South Dakota and North Dakota, among other states, with very low coefficients, similar to the findings in this paper.

The average of the posterior mean of the sum of the Okun's law coefficients across states is about -0.3, which is smaller in absolute value than the usual coefficient of -0.5. Likewise, Owyang and Sekhposyan (2012) and Grant (2018) report a statistically significant decline in the reaction of the unemployment gap to the output gap during the Great Recession. Similarly, Ball, Leigh and Loungani (2017) also find that the Okun's law coefficient has declined recently, but the difference with respect to the past does not appear to be statistically significant.

Table M.14 shows that those states with higher participation of the construction industry tend to be characterized by more-cyclical labor markets in this period, whereas those states that have higher contributions from agriculture and mining tend to have less-cyclical labor markets, as is expected from these sectors. Guisinger et al. (2018) emphasize that institutional differences can help explain the variation in the size of Okun's law coefficients across states. One of those institutional factors is the percentage of the workforce who are union members. I add said variable to the set of regressors previously used, but the results show that this variable is not statistically significant at conventional levels.

As a final exercise with respect to labor market cyclicality, the model allows one to estimate the effect of an increase in the aggregate output gap on each state's unemployment rate. This exercise is particularly useful to understand how the unemployment rate reacts at the state level with respect to policies designed at the aggregate level. Figure M.7 shows the response of the state-level unemployment rate to such a shock over 20 quarters for the three most sensitive and the three least sensitive states.[3] On the one hand, a 0.8 percentage point increase in the aggregate output gap causes the unemployment rates of Nevada, Arizona, and California to decline by about 1, 0.8, and 0.75 percentage point, respectively, within 3 quarters of the shock. On the other hand, the responses of Wyoming, North Dakota, and West Virginia imply a muted effect on the unemployment rates of those states.

K Variance Decomposition of Aggregate GDP Growth

As described in Section 2, the common trend and cycle components imply co-movement among the state economies that, in turn, influence the variability of the aggregate GDP. One can compute the variance decomposition of aggregate GDP growth, which provides an indicator of how cyclical is the U.S. economy, as follows:[4]

$$\text{var}(\Delta\% \text{ Aggregate GDP}) = \text{var}(\Delta \text{ Aggregate GDP trend}) + \text{var}(\Delta \text{ Output Gap}).$$

[3]The state-level unemployment rate trend and the idiosyncratic component of the state's cycle remain constant in this exercise.

[4]The full derivation appears in online Appendix L.

Therefore, the percent of the variance of GDP growth that is due to the variance of the change in the aggregate cycle is given by the following expression:

$$\frac{\text{var}(\Delta \text{ Output Gap})}{\text{var}(\Delta \text{ Aggregate GDP trend}) + \text{var}(\Delta \text{ Output Gap})} = 62\%.$$

The contribution of the variability of the cycle to the variability of GDP growth of 62 percent is close to estimates obtained with longer samples. For example, Gonzalez-Astudillo and Roberts (2016) find that the contribution is around 60 percent or 65 percent, depending on the assumption about the correlation between trend and cycle components.

L Derivation of the Variance Decomposition of Aggregate GDP

Given that (the log of) aggregate GDP can be specified as the sum of its aggregate trend and cycle, one can write the following:

$$\begin{aligned}
\text{var}(\Delta\%Y_t) &= \text{var}(\Delta \text{ Aggregate GDP trend}) + \text{var}(\Delta \text{ Output Gap}) \\
&\approx \text{var}(\bar{\delta^y}\Delta\tau_t + \Delta\bar{\xi}_t^y) + \text{var}(\bar{\alpha}\Delta c_t + \Delta\bar{\upsilon}_t^y) \\
&= \bar{\delta^y}^2 \,\text{var}(\eta_t^y + \bar{\eta^y}_t) + \bar{\alpha}^2 \,\text{var}(\Delta c_t + \Delta\bar{\upsilon}_t^y) \\
&\approx \bar{\delta^y}^2 \left(1 + \sum_{i=1}^{n} w_i^2 \sigma_{\eta_i^y}^2\right) + \bar{\alpha}^2 \left(2(\gamma_0 - \gamma_1) + \sum_{i=1}^{n} w_i^2 \sigma_{\Delta\upsilon_{it}}^2\right),
\end{aligned}$$

where

$$\begin{aligned}
\gamma_0 &= \frac{\frac{1-\phi_2}{1+\phi_2}}{(1-\phi_2^2)-\phi_1^2}, \\
\gamma_1 &= \frac{\phi_1}{1-\phi_2}\gamma_0.
\end{aligned}$$

M Prior and Posterior Distribution Results from the Aggregate UC Model

The prior distribution choices and the posterior mean and standard deviation results appear in table M.12.

[Table 12 about here.]

The mean annual growth rate of potential GDP (4μ) is estimated to be close to 1.6 percent, 1 percentage point lower than the estimate with state-level data obtained from weight averaging the estimated state-level mean growth rates ($4\sum_{i=1}^{51} w_i\mu_i$). With respect to the Okun's law coefficients, the long-run sensitivity of the unemployment gap to the output gap is about -0.59, more than twice the simple average obtained from the state-level data. Owyang, Vermann and Sekhposyan (2013) also investigate the Okun's law coefficients across states, although they use the growth rates version of the law, and find that there can be discrepancies between the estimates with state-level data and the estimate obtained from aggregate data.

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Table M.1: Prior Distributions of the Parameters using State-Level Data

Parameter	Distribution	Mean	Standard Deviation	Equation
ϕ_1	Truncated Normal	1.5	1	$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \varepsilon_t$
ϕ_2	Truncated Normal	-0.6	1	$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \varepsilon_t$
μ_i	Truncated Normal	0.8	0.8	$\xi^y_{it} = \mu_i + \xi^y_{i,t-1} + \eta^y_{it}$
α_i	Normal	1	1	$c_{it} = \alpha_i c_t + \upsilon_{it}$
δ^y_i	Truncated Normal	1	1	$\tau^y_{it} = \delta^y_i \tau^y_t + \xi^y_{it}$
δ^u_i	Truncated Normal	1	1	$\tau^u_{it} = \delta^u_i \tau^u_t + \xi^u_{it}$
θ_{1i}	Normal	-0.25	0.25	$u_{it} = \tau^u_{it} + \theta_{1i} c_{it} + \theta_{2i} c_{i,t-1}$
θ_{2i}	Normal	-0.25	0.25	$u_{it} = \tau^u_{it} + \theta_{1i} c_{it} + \theta_{2i} c_{i,t-1}$
ρ_i	Truncated Normal	0	1	$\upsilon_{it} = \rho_i \upsilon_{i,t-1} + \zeta_{it}$
$\sigma^2_{\eta^y_i}$	Inverse Gamma	1	Inf	$\xi^y_{it} = \mu_i + \xi^y_{i,t-1} + \eta^y_{it}$, $\mathrm{var}(\eta^y_{it}) = \sigma^2_{\eta^y_i}$
$\sigma^2_{\eta^u_i}$	Inverse Gamma	1	Inf	$\xi^u_{it} = \xi^u_{i,t-1} + \eta^u_{it}$, $\mathrm{var}(\eta^u_{it}) = \sigma^2_{\eta^u_i}$
$\sigma^2_{\zeta_i}$	Inverse Gamma	1	Inf	$\upsilon_{it} = \rho_i \upsilon_{i,t-1} + \zeta_{it}$, $\mathrm{var}(\zeta_{it}) = \sigma^2_{\zeta_i}$

Table M.2: Posterior Distribution Estimates and Convergence Diagnostics

Parameter	Mean	Sd.	auto1	auto50	RNE	p-value
ϕ_1	1.518	0.159	0.026	0.014	0.840	0.923
ϕ_2	-0.555	0.157	0.025	0.014	0.813	0.783
c_T	0.920	10.458	0.005	0.004	0.868	0.587
μ_1	0.553	0.135	0.182	0.006	0.401	0.946
μ_2	0.524	0.272	0.165	-0.025	0.309	0.548
μ_3	0.676	0.148	0.101	0.015	0.597	0.973
μ_4	0.676	0.190	0.075	0.011	0.680	0.567
μ_5	0.744	0.123	0.049	-0.015	0.542	0.830
μ_6	0.757	0.119	0.077	0.022	0.833	0.787
μ_7	0.271	0.157	0.010	-0.023	0.898	0.235
μ_8	0.472	0.275	-0.010	-0.012	1.043	0.844
μ_9	0.564	0.116	0.019	-0.003	0.718	0.058
μ_{10}	0.570	0.133	0.153	0.018	0.497	0.101
μ_{11}	0.570	0.121	0.130	-0.009	0.353	0.744
μ_{12}	0.606	0.115	0.159	-0.000	0.470	0.688
μ_{13}	0.749	0.149	0.048	0.034	0.777	0.245
μ_{14}	0.485	0.114	0.122	-0.013	0.612	0.615
μ_{15}	0.827	0.193	0.253	0.019	0.202	0.425
μ_{16}	0.699	0.181	0.082	0.014	0.495	0.867
μ_{17}	0.843	0.223	0.049	-0.018	0.911	0.057
μ_{18}	0.749	0.176	0.196	0.004	0.303	0.370
μ_{19}	0.235	0.148	0.025	-0.016	1.441	0.187
μ_{20}	0.464	0.139	0.164	-0.015	0.343	0.935
μ_{21}	0.639	0.114	0.163	0.034	0.331	0.886
μ_{22}	0.799	0.124	0.124	-0.012	0.502	0.528
μ_{23}	0.521	0.197	0.188	-0.006	0.320	0.444
μ_{24}	0.634	0.153	0.173	-0.005	0.228	0.532
μ_{25}	0.564	0.185	0.112	-0.013	0.457	0.616
μ_{26}	0.453	0.132	0.054	-0.014	0.886	0.342
μ_{27}	0.549	0.130	0.031	-0.003	0.813	0.431
μ_{28}	0.762	0.162	0.124	0.001	0.340	0.206
μ_{29}	0.541	0.166	0.106	-0.005	0.808	0.078
μ_{30}	0.584	0.145	0.018	0.025	0.661	0.660
μ_{31}	0.524	0.133	0.132	0.014	0.507	0.780
μ_{32}	0.372	0.179	0.152	-0.011	0.259	0.293
μ_{33}	0.507	0.188	-0.001	-0.027	0.837	0.123
μ_{34}	0.675	0.160	0.071	0.020	0.406	0.309
μ_{35}	1.309	0.307	0.008	0.026	0.881	0.283
μ_{36}	0.542	0.151	0.184	0.023	0.279	0.746
μ_{37}	0.839	0.276	0.057	-0.023	0.338	0.778
μ_{38}	0.820	0.152	0.029	-0.010	0.958	0.702
μ_{39}	0.741	0.137	0.208	0.033	0.226	0.838
μ_{40}	0.383	0.161	0.071	0.022	0.683	0.072
μ_{41}	0.720	0.124	0.128	0.011	0.358	0.743
μ_{42}	0.689	0.237	0.016	-0.014	0.756	0.631
μ_{43}	0.682	0.129	0.155	0.017	0.546	0.231
μ_{44}	0.853	0.136	0.088	0.017	0.449	0.505
μ_{45}	0.877	0.126	0.045	0.018	1.111	0.072
μ_{46}	0.594	0.151	0.100	-0.013	0.575	0.691
μ_{47}	0.527	0.104	0.117	0.055	0.545	0.326
μ_{48}	1.053	0.140	0.084	0.003	0.482	0.752
μ_{49}	0.521	0.159	0.110	-0.010	0.874	0.850
μ_{50}	0.553	0.118	0.145	0.004	0.375	0.968
μ_{51}	0.614	0.306	0.114	-0.001	0.256	0.765

Note: Statistics are based on 2,000 draws from the Gibbs sampler. Mean: average across draws; Sd.: standard deviation across draws; auto1: first order autocorrelation across draws; auto50: fiftieth order autocorrelation across draws; RNE: relative numerical efficiency using a 4 percent taper; p-value: p-value of the null hypothesis that the mean of the first 20 percent of draws is equal to the mean of the last 50 percent of draws using a 4 percent taper for the standard error.

Table M.3: Posterior Distribution Estimates and Convergence Diagnostics

Parameter	Mean	Sd.	auto1	auto50	RNE	p-value
α_{1}	0.712	0.252	0.231	-0.009	0.195	0.476
α_{2}	-0.420	0.570	0.085	-0.018	0.407	0.811
α_{3}	1.728	0.303	0.035	0.016	1.015	0.950
α_{4}	0.766	0.338	0.161	0.012	0.231	0.637
α_{5}	1.601	0.313	0.103	-0.013	0.277	0.811
α_{6}	1.036	0.254	0.045	0.035	0.540	0.424
α_{7}	1.005	0.318	0.027	0.012	0.783	0.784
α_{8}	1.073	0.432	-0.012	0.028	0.795	0.904
α_{9}	0.235	0.210	0.107	-0.027	0.393	0.212
α_{10}	1.642	0.285	0.038	0.012	0.517	0.703
α_{11}	1.316	0.235	0.021	0.009	0.792	0.894
α_{12}	0.813	0.200	0.079	-0.027	0.477	0.379
α_{13}	1.160	0.276	0.020	0.002	0.964	0.025
α_{14}	0.891	0.215	0.075	0.002	0.635	0.389
α_{15}	1.162	0.349	0.243	-0.016	0.180	0.865
α_{16}	0.855	0.336	0.089	-0.005	0.398	0.221
α_{17}	0.957	0.405	0.063	0.008	0.471	0.735
α_{18}	0.797	0.327	0.314	-0.026	0.143	0.903
α_{19}	0.060	0.326	0.028	-0.002	0.571	0.851
α_{20}	0.517	0.262	0.224	-0.030	0.228	0.829
α_{21}	0.503	0.207	0.104	-0.007	0.257	0.677
α_{22}	0.666	0.212	0.137	0.025	0.364	0.987
α_{23}	1.074	0.456	0.152	-0.023	0.238	0.900
α_{24}	0.814	0.295	0.107	-0.010	0.344	0.954
α_{25}	0.547	0.304	0.113	-0.028	0.251	0.695
α_{26}	0.219	0.247	0.214	-0.026	0.166	0.969
α_{27}	0.722	0.247	-0.004	0.009	0.894	0.443
α_{28}	0.478	0.298	0.063	-0.013	0.442	0.718
α_{29}	1.836	0.331	0.044	0.011	0.756	0.655
α_{30}	0.674	0.271	0.099	-0.033	0.393	0.350
α_{31}	0.834	0.231	0.118	0.004	0.362	0.641
α_{32}	0.314	0.298	0.024	0.044	0.790	0.700
α_{33}	0.631	0.332	-0.020	-0.039	1.496	0.711
α_{34}	1.090	0.295	0.020	-0.021	0.556	0.084
α_{35}	-0.083	0.469	0.084	-0.011	0.318	0.830
α_{36}	0.917	0.277	0.129	0.019	0.327	0.055
α_{37}	0.474	0.483	0.006	0.025	1.071	0.848
α_{38}	1.307	0.292	0.010	0.022	0.854	0.742
α_{39}	0.667	0.244	0.191	-0.014	0.211	0.790
α_{40}	0.570	0.292	0.046	-0.039	0.682	0.865
α_{41}	1.246	0.238	0.056	-0.028	0.807	0.822
α_{42}	0.318	0.438	0.064	0.004	0.541	0.173
α_{43}	0.958	0.234	0.026	0.014	0.694	0.213
α_{44}	0.737	0.300	0.057	0.006	0.409	0.516
α_{45}	1.301	0.311	0.058	0.073	0.502	0.960
α_{46}	0.156	0.312	0.313	-0.001	0.128	0.637
α_{47}	0.454	0.191	0.113	0.005	0.266	0.124
α_{48}	1.185	0.263	-0.010	0.012	1.784	0.823
α_{49}	-0.022	0.295	0.203	0.011	0.221	0.870
α_{50}	0.813	0.219	0.094	-0.054	0.399	0.304
α_{51}	0.056	0.501	0.056	0.025	1.122	0.344

Note: Statistics are based on 2,000 draws from the Gibbs sampler. Mean: average across draws; Sd.: standard deviation across draws; auto1: first order autocorrelation across draws; auto50: fiftieth order autocorrelation across draws; RNE: relative numerical efficiency using a 4 percent taper; p-value: p-value of the null hypothesis that the mean of the first 20 percent of draws is equal to the mean of the last 50 percent of draws using a 4 percent taper for the standard error.

Table M.4: Posterior Distribution Estimates and Convergence Diagnostics

Parameter	Mean	Sd.	auto1	auto50	RNE	p-value
δ_{1}^{y}	1.391	0.242	0.252	-0.053	0.190	0.868
δ_{2}^{y}	0.563	0.558	0.527	-0.031	0.078	0.854
δ_{3}^{y}	1.254	0.279	0.050	0.012	0.520	0.604
δ_{4}^{y}	1.637	0.347	0.012	0.030	0.574	0.960
δ_{5}^{y}	0.637	0.264	0.030	-0.017	1.312	0.533
δ_{6}^{y}	0.792	0.240	0.046	0.006	0.773	0.990
δ_{7}^{y}	0.841	0.330	0.015	-0.032	0.906	0.632
δ_{8}^{y}	0.647	0.434	-0.004	0.034	0.951	0.013
δ_{9}^{y}	0.529	0.238	0.052	-0.015	0.454	0.845
δ_{10}^{y}	1.134	0.249	0.095	0.040	0.501	0.405
δ_{11}^{y}	0.944	0.242	0.161	-0.031	0.238	0.881
δ_{12}^{y}	1.056	0.210	0.147	-0.057	0.289	0.530
δ_{13}^{y}	1.081	0.282	-0.009	-0.002	1.377	0.994
δ_{14}^{y}	1.038	0.214	0.100	-0.030	0.453	0.871
δ_{15}^{y}	1.992	0.366	0.363	-0.013	0.113	0.730
δ_{16}^{y}	1.367	0.341	0.144	-0.042	0.284	0.539
δ_{17}^{y}	1.600	0.418	0.022	0.017	0.847	0.064
δ_{18}^{y}	1.932	0.306	0.256	-0.044	0.167	0.265
δ_{19}^{y}	1.163	0.329	-0.024	-0.013	0.819	0.332
δ_{20}^{y}	1.294	0.257	0.217	-0.034	0.169	0.804
δ_{21}^{y}	0.911	0.234	0.217	-0.045	0.188	0.949
δ_{22}^{y}	1.098	0.233	0.158	-0.049	0.334	0.491
δ_{23}^{y}	1.570	0.420	0.233	-0.016	0.171	0.822
δ_{24}^{y}	1.228	0.310	0.314	-0.015	0.125	0.478
δ_{25}^{y}	1.516	0.349	0.162	0.008	0.240	0.418
δ_{26}^{y}	1.103	0.247	0.018	-0.034	1.078	0.178
δ_{27}^{y}	0.513	0.254	-0.007	0.012	0.883	0.408
δ_{28}^{y}	1.021	0.357	0.237	-0.058	0.164	0.215
δ_{29}^{y}	1.476	0.299	0.024	-0.008	1.421	0.224
δ_{30}^{y}	0.868	0.303	0.049	-0.029	0.650	0.533
δ_{31}^{y}	1.294	0.240	0.186	-0.016	0.227	0.546
δ_{32}^{y}	0.667	0.427	0.403	-0.025	0.101	0.596
δ_{33}^{y}	0.251	0.210	-0.019	-0.017	0.988	0.260
δ_{34}^{y}	1.039	0.313	0.116	-0.017	0.292	0.632
δ_{35}^{y}	0.911	0.496	-0.017	-0.018	0.845	0.687
δ_{36}^{y}	1.247	0.302	0.303	-0.015	0.133	0.362
δ_{37}^{y}	0.873	0.541	0.290	-0.003	0.140	0.790
δ_{38}^{y}	0.814	0.296	0.060	-0.019	0.745	0.242
δ_{39}^{y}	1.304	0.265	0.298	-0.029	0.145	0.746
δ_{40}^{y}	1.066	0.307	0.055	0.026	0.538	0.445
δ_{41}^{y}	1.214	0.236	0.181	-0.014	0.263	0.931
δ_{42}^{y}	0.693	0.405	0.000	0.005	0.618	0.505
δ_{43}^{y}	1.062	0.240	0.166	0.012	0.290	0.935
δ_{44}^{y}	0.396	0.281	0.303	-0.018	0.124	0.627
δ_{45}^{y}	0.567	0.249	0.053	0.003	0.768	0.055
δ_{46}^{y}	1.415	0.285	0.108	-0.032	0.311	0.591
δ_{47}^{y}	0.925	0.202	0.155	0.004	0.305	0.482
δ_{48}^{y}	1.132	0.262	0.061	0.019	0.465	0.564
δ_{49}^{y}	1.234	0.324	0.112	-0.025	0.361	0.889
δ_{50}^{y}	1.072	0.227	0.140	0.016	0.272	0.804
δ_{51}^{y}	0.774	0.703	0.543	-0.014	0.073	0.914

Note: Statistics are based on 2,000 draws from the Gibbs sampler. Mean: average across draws; Sd.: standard deviation across draws; auto1: first order autocorrelation across draws; auto50: fiftieth order autocorrelation across draws; RNE: relative numerical efficiency using a 4 percent taper; p-value: p-value of the null hypothesis that the mean of the first 20 percent of draws is equal to the mean of the last 50 percent of draws using a 4 percent taper for the standard error.

Table M.5: Posterior Distribution Estimates and Convergence Diagnostics

Parameter	Mean	Sd.	auto1	auto50	RNE	p-value
δ_{1}^{u}	1.601	0.272	0.079	-0.031	0.957	0.413
δ_{2}^{u}	0.406	0.148	0.008	-0.043	1.292	0.946
δ_{3}^{u}	0.732	0.227	0.184	-0.033	0.242	0.932
δ_{4}^{u}	0.747	0.186	0.109	0.013	0.425	0.807
δ_{5}^{u}	0.975	0.212	0.101	-0.017	0.366	0.629
δ_{6}^{u}	0.619	0.199	0.121	-0.044	0.347	0.972
δ_{7}^{u}	0.511	0.191	0.127	0.011	0.243	0.718
δ_{8}^{u}	0.844	0.214	0.036	-0.020	1.181	0.199
δ_{9}^{u}	0.812	0.215	0.045	0.012	0.826	0.884
δ_{10}^{u}	1.083	0.219	0.036	0.018	0.641	0.495
δ_{11}^{u}	0.941	0.202	0.103	-0.014	0.426	0.664
δ_{12}^{u}	0.680	0.192	0.039	-0.029	0.888	0.338
δ_{13}^{u}	0.854	0.260	0.097	0.001	0.367	0.661
δ_{14}^{u}	1.187	0.242	0.043	-0.005	0.752	0.426
δ_{15}^{u}	1.229	0.248	0.078	-0.046	0.763	0.654
δ_{16}^{u}	0.649	0.170	0.068	-0.008	0.534	0.694
δ_{17}^{u}	0.709	0.163	0.068	-0.031	0.669	0.993
δ_{18}^{u}	1.184	0.236	0.078	-0.019	0.764	0.386
δ_{19}^{u}	0.784	0.400	0.040	0.032	0.488	0.298
δ_{20}^{u}	0.805	0.190	0.095	-0.023	0.608	0.693
δ_{21}^{u}	0.765	0.181	0.029	-0.005	0.999	0.576
δ_{22}^{u}	0.679	0.173	0.033	-0.016	0.854	0.901
δ_{23}^{u}	1.679	0.339	0.140	-0.012	0.326	0.621
δ_{24}^{u}	0.785	0.181	0.066	-0.032	0.725	0.544
δ_{25}^{u}	0.831	0.277	0.135	0.007	0.378	0.779
δ_{26}^{u}	1.117	0.210	0.098	-0.019	0.570	0.687
δ_{27}^{u}	0.499	0.176	0.015	-0.014	0.569	0.116
δ_{28}^{u}	0.460	0.131	0.029	-0.047	0.885	0.488
δ_{29}^{u}	1.003	0.270	0.211	-0.040	0.198	0.758
δ_{30}^{u}	0.705	0.182	0.108	-0.005	0.510	0.508
δ_{31}^{u}	0.904	0.216	0.026	0.012	0.751	0.953
δ_{32}^{u}	0.908	0.215	0.075	0.005	0.836	0.257
δ_{33}^{u}	0.803	0.201	0.056	-0.021	0.716	0.520
δ_{34}^{u}	1.257	0.243	0.089	-0.018	0.701	0.613
δ_{35}^{u}	0.391	0.151	0.060	-0.038	0.432	0.550
δ_{36}^{u}	1.223	0.232	0.073	-0.016	0.537	0.745
δ_{37}^{u}	0.819	0.197	0.080	0.008	0.477	0.784
δ_{38}^{u}	1.307	0.289	0.218	0.018	0.242	0.971
δ_{39}^{u}	0.831	0.183	0.042	-0.019	0.790	0.286
δ_{40}^{u}	0.974	0.245	0.042	0.026	0.938	0.515
δ_{41}^{u}	1.198	0.241	0.070	0.031	0.845	0.253
δ_{42}^{u}	0.582	0.162	0.041	-0.052	0.788	0.788
δ_{43}^{u}	1.279	0.256	0.025	0.018	0.764	0.897
δ_{44}^{u}	0.766	0.191	0.078	-0.023	0.619	0.151
δ_{45}^{u}	1.006	0.216	0.077	-0.021	0.979	0.393
δ_{46}^{u}	0.647	0.171	0.061	-0.028	0.423	0.870
δ_{47}^{u}	0.863	0.171	0.051	-0.066	0.625	0.719
δ_{48}^{u}	0.946	0.223	0.120	-0.025	0.420	0.683
δ_{49}^{u}	1.144	0.242	0.096	-0.044	0.623	0.543
δ_{50}^{u}	1.115	0.216	0.080	-0.009	0.569	0.549
δ_{51}^{u}	1.078	0.252	0.057	-0.046	0.957	0.191

Note: Statistics are based on 2,000 draws from the Gibbs sampler. Mean: average across draws; Sd.: standard deviation across draws; auto1: first order autocorrelation across draws; auto50: fiftieth order autocorrelation across draws; RNE: relative numerical efficiency using a 4 percent taper; p-value: p-value of the null hypothesis that the mean of the first 20 percent of draws is equal to the mean of the last 50 percent of draws using a 4 percent taper for the standard error.

Table M.6: Posterior Distribution Estimates and Convergence Diagnostics

Parameter	Mean	Sd.	auto1	auto50	RNE	p-value
$\theta_{1,1}$	-0.299	0.078	-0.002	-0.020	0.906	0.884
$\theta_{1,2}$	-0.092	0.064	0.094	0.007	0.577	0.494
$\theta_{1,3}$	-0.186	0.046	0.018	-0.038	0.795	0.134
$\theta_{1,4}$	-0.094	0.050	-0.015	-0.004	1.109	0.176
$\theta_{1,5}$	-0.162	0.042	-0.043	-0.022	1.397	0.339
$\theta_{1,6}$	-0.212	0.059	0.013	-0.032	0.620	0.629
$\theta_{1,7}$	-0.167	0.062	0.029	-0.029	0.714	0.645
$\theta_{1,8}$	-0.165	0.066	0.051	0.005	0.650	0.252
$\theta_{1,9}$	-0.250	0.076	-0.055	-0.026	0.861	0.282
$\theta_{1,10}$	-0.187	0.047	0.072	0.003	0.773	0.422
$\theta_{1,11}$	-0.211	0.050	-0.012	0.033	1.168	0.449
$\theta_{1,12}$	-0.251	0.066	0.014	-0.021	0.720	0.191
$\theta_{1,13}$	-0.209	0.073	0.034	0.047	0.439	0.127
$\theta_{1,14}$	-0.247	0.069	0.028	0.001	0.961	0.974
$\theta_{1,15}$	-0.222	0.062	0.003	-0.018	0.506	0.630
$\theta_{1,16}$	-0.111	0.059	-0.001	0.013	1.252	0.512
$\theta_{1,17}$	-0.078	0.048	0.002	-0.001	0.876	0.777
$\theta_{1,18}$	-0.199	0.073	0.022	-0.022	0.767	0.787
$\theta_{1,19}$	-0.382	0.101	0.035	0.029	0.534	0.945
$\theta_{1,20}$	-0.235	0.070	-0.016	0.006	1.052	0.806
$\theta_{1,21}$	-0.229	0.068	-0.014	0.021	0.879	0.918
$\theta_{1,22}$	-0.196	0.060	0.042	0.019	0.552	0.611
$\theta_{1,23}$	-0.176	0.072	0.022	0.038	0.813	0.938
$\theta_{1,24}$	-0.114	0.056	0.007	0.030	0.872	0.462
$\theta_{1,25}$	-0.238	0.093	0.129	-0.002	0.322	0.905
$\theta_{1,26}$	-0.193	0.070	0.022	0.055	0.755	0.403
$\theta_{1,27}$	-0.196	0.063	-0.022	0.012	1.115	0.456
$\theta_{1,28}$	-0.057	0.036	-0.031	-0.011	0.879	0.866
$\theta_{1,29}$	-0.207	0.054	-0.012	-0.029	0.715	0.866
$\theta_{1,30}$	-0.093	0.050	0.024	-0.024	0.572	0.701
$\theta_{1,31}$	-0.237	0.069	-0.005	0.045	0.446	0.898
$\theta_{1,32}$	-0.200	0.081	0.043	-0.006	0.465	0.391
$\theta_{1,33}$	-0.180	0.072	-0.011	-0.004	0.694	0.353
$\theta_{1,34}$	-0.231	0.063	-0.005	-0.026	0.630	0.382
$\theta_{1,35}$	-0.107	0.064	0.007	0.044	1.415	0.312
$\theta_{1,36}$	-0.165	0.067	-0.021	0.019	0.921	0.107
$\theta_{1,37}$	-0.107	0.069	0.006	-0.013	0.781	0.264
$\theta_{1,38}$	-0.209	0.067	0.097	0.045	0.322	0.467
$\theta_{1,39}$	-0.198	0.065	0.028	0.012	0.841	0.914
$\theta_{1,40}$	-0.267	0.074	0.002	-0.006	1.039	0.625
$\theta_{1,41}$	-0.228	0.059	-0.006	-0.010	1.144	0.417
$\theta_{1,42}$	-0.082	0.060	0.072	-0.005	0.865	0.863
$\theta_{1,43}$	-0.205	0.073	-0.018	0.034	1.385	0.123
$\theta_{1,44}$	-0.170	0.065	-0.011	0.018	0.891	0.152
$\theta_{1,45}$	-0.143	0.055	0.017	-0.055	0.777	0.524
$\theta_{1,46}$	-0.160	0.068	-0.015	-0.020	1.011	0.378
$\theta_{1,47}$	-0.222	0.066	-0.011	0.015	1.270	0.854
$\theta_{1,48}$	-0.172	0.060	0.043	-0.040	0.794	0.212
$\theta_{1,49}$	-0.237	0.080	0.032	-0.005	1.093	0.227
$\theta_{1,50}$	-0.168	0.065	-0.006	-0.013	0.732	0.469
$\theta_{1,51}$	-0.189	0.081	0.019	0.036	1.126	0.557

Note: Statistics are based on 2,000 draws from the Gibbs sampler. Mean: average across draws; Sd.: standard deviation across draws; auto1: first order autocorrelation across draws; auto50: fiftieth order autocorrelation across draws; RNE: relative numerical efficiency using a 4 percent taper; p-value: p-value of the null hypothesis that the mean of the first 20 percent of draws is equal to the mean of the last 50 percent of draws using a 4 percent taper for the standard error.

Table M.7: Posterior Distribution Estimates and Convergence Diagnostics

Parameter	Mean	Sd.	auto1	auto50	RNE	p-value
$\theta_{2,1}$	-0.080	0.074	0.020	0.032	1.039	0.817
$\theta_{2,2}$	-0.035	0.049	0.069	0.027	0.605	0.615
$\theta_{2,3}$	-0.087	0.048	0.113	0.017	0.304	0.765
$\theta_{2,4}$	-0.069	0.045	0.012	-0.021	1.387	0.716
$\theta_{2,5}$	-0.100	0.043	0.064	-0.043	0.632	0.248
$\theta_{2,6}$	-0.123	0.058	0.075	0.012	0.387	0.539
$\theta_{2,7}$	-0.110	0.056	0.084	0.020	0.282	0.254
$\theta_{2,8}$	-0.095	0.051	0.006	-0.009	0.607	0.381
$\theta_{2,9}$	-0.146	0.074	-0.011	0.004	1.139	0.314
$\theta_{2,10}$	-0.057	0.046	0.007	0.019	0.911	0.307
$\theta_{2,11}$	-0.083	0.051	0.033	-0.059	0.928	0.544
$\theta_{2,12}$	-0.103	0.064	0.018	-0.006	0.637	0.361
$\theta_{2,13}$	-0.145	0.070	0.021	-0.002	0.607	0.895
$\theta_{2,14}$	-0.084	0.067	-0.029	0.032	1.294	0.826
$\theta_{2,15}$	-0.072	0.062	0.018	0.038	1.021	0.303
$\theta_{2,16}$	-0.041	0.050	0.012	-0.027	0.656	0.525
$\theta_{2,17}$	-0.043	0.039	0.024	0.041	1.395	0.580
$\theta_{2,18}$	-0.093	0.068	0.041	-0.005	1.293	0.718
$\theta_{2,19}$	-0.065	0.098	-0.008	0.025	1.488	0.950
$\theta_{2,20}$	-0.091	0.066	-0.046	0.019	0.818	0.270
$\theta_{2,21}$	-0.113	0.067	0.045	-0.001	0.698	0.676
$\theta_{2,22}$	-0.122	0.056	0.010	0.014	0.643	0.488
$\theta_{2,23}$	-0.098	0.069	0.077	0.001	0.439	0.156
$\theta_{2,24}$	-0.056	0.053	0.004	-0.015	1.129	0.466
$\theta_{2,25}$	-0.186	0.085	0.084	0.008	0.361	0.824
$\theta_{2,26}$	-0.113	0.065	-0.022	0.001	1.574	0.639
$\theta_{2,27}$	-0.079	0.060	0.009	-0.003	0.885	0.444
$\theta_{2,28}$	-0.033	0.035	-0.001	0.025	0.716	0.654
$\theta_{2,29}$	-0.086	0.056	0.170	-0.023	0.277	0.911
$\theta_{2,30}$	-0.073	0.048	0.037	-0.000	0.564	0.647
$\theta_{2,31}$	-0.121	0.066	0.064	0.001	0.838	0.336
$\theta_{2,32}$	-0.087	0.070	0.070	0.051	0.851	0.386
$\theta_{2,33}$	-0.088	0.061	-0.020	0.004	1.060	0.007
$\theta_{2,34}$	-0.042	0.059	0.003	0.022	0.783	0.059
$\theta_{2,35}$	-0.040	0.054	0.000	0.009	0.911	0.337
$\theta_{2,36}$	-0.071	0.062	0.037	0.008	0.751	0.428
$\theta_{2,37}$	-0.055	0.053	-0.000	0.031	0.510	0.024
$\theta_{2,38}$	-0.075	0.065	0.159	0.003	0.274	0.713
$\theta_{2,39}$	-0.075	0.060	-0.052	0.027	1.149	0.369
$\theta_{2,40}$	-0.165	0.063	0.035	0.022	0.942	0.644
$\theta_{2,41}$	-0.077	0.058	0.044	0.021	0.798	0.067
$\theta_{2,42}$	-0.048	0.046	0.032	-0.003	1.052	0.980
$\theta_{2,43}$	-0.046	0.070	0.007	0.035	0.858	0.214
$\theta_{2,44}$	-0.083	0.056	0.020	-0.033	0.952	0.575
$\theta_{2,45}$	-0.097	0.052	0.066	0.007	0.585	0.919
$\theta_{2,46}$	-0.095	0.064	-0.007	0.031	1.171	0.967
$\theta_{2,47}$	-0.098	0.063	0.025	-0.017	1.104	0.542
$\theta_{2,48}$	-0.068	0.054	-0.030	0.020	0.926	0.684
$\theta_{2,49}$	-0.062	0.077	0.073	0.016	0.497	0.350
$\theta_{2,50}$	-0.061	0.063	0.045	-0.000	0.525	0.035
$\theta_{2,51}$	-0.078	0.068	0.073	-0.031	0.702	0.255

Note: Statistics are based on 2,000 draws from the Gibbs sampler. Mean: average across draws; Sd.: standard deviation across draws; auto1: first order autocorrelation across draws; auto50: fiftieth order autocorrelation across draws; RNE: relative numerical efficiency using a 4 percent taper; p-value: p-value of the null hypothesis that the mean of the first 20 percent of draws is equal to the mean of the last 50 percent of draws using a 4 percent taper for the standard error.

Table M.8: Posterior Distribution Estimates and Convergence Diagnostics

Parameter	Mean	Sd.	auto1	auto50	RNE	p-value
ρ_1	0.749	0.210	-0.006	0.035	0.987	0.409
ρ_2	0.616	0.299	-0.011	-0.019	1.043	0.428
ρ_3	0.702	0.308	0.052	-0.017	0.713	0.755
ρ_4	-0.108	0.321	0.016	-0.027	1.026	0.613
ρ_5	0.477	0.352	-0.025	0.020	1.024	0.323
ρ_6	0.706	0.304	0.005	0.052	1.021	0.400
ρ_7	0.253	0.382	-0.009	0.011	1.389	0.013
ρ_8	0.253	0.437	0.016	-0.027	0.956	0.663
ρ_9	0.141	0.399	0.025	-0.009	0.870	0.052
ρ_{10}	0.818	0.203	0.013	-0.008	0.880	0.607
ρ_{11}	0.578	0.326	0.014	-0.026	1.044	0.803
ρ_{12}	0.741	0.201	0.015	-0.010	1.225	0.978
ρ_{13}	0.344	0.338	0.058	0.004	0.978	0.670
ρ_{14}	0.704	0.241	0.013	0.024	0.667	0.741
ρ_{15}	0.790	0.171	0.018	0.009	1.295	0.122
ρ_{16}	0.729	0.217	-0.007	-0.048	1.246	0.936
ρ_{17}	0.230	0.358	-0.006	0.032	1.259	0.086
ρ_{18}	0.599	0.286	-0.075	-0.039	1.398	0.551
ρ_{19}	0.732	0.230	0.028	0.011	0.656	0.095
ρ_{20}	0.717	0.288	-0.019	0.004	1.368	0.559
ρ_{21}	0.673	0.282	0.031	0.007	1.832	0.899
ρ_{22}	0.412	0.339	0.008	-0.014	1.099	0.975
ρ_{23}	0.702	0.332	-0.012	-0.002	1.346	0.816
ρ_{24}	0.421	0.375	0.004	-0.014	1.100	0.218
ρ_{25}	0.498	0.377	-0.017	0.022	1.022	0.708
ρ_{26}	0.081	0.400	0.014	0.011	0.608	0.618
ρ_{27}	0.583	0.327	-0.013	-0.023	1.015	0.596
ρ_{28}	-0.130	0.266	0.042	0.007	1.243	0.687
ρ_{29}	0.522	0.325	0.048	-0.039	0.551	0.378
ρ_{30}	-0.390	0.252	0.007	0.009	0.903	0.897
ρ_{31}	0.581	0.325	0.042	0.012	0.931	0.199
ρ_{32}	0.507	0.322	0.005	0.011	1.088	0.404
ρ_{33}	0.466	0.367	-0.030	-0.033	0.896	0.279
ρ_{34}	0.548	0.263	-0.003	0.009	1.217	0.509
ρ_{35}	0.595	0.369	0.049	0.019	0.978	0.740
ρ_{36}	0.750	0.240	-0.007	0.016	1.187	0.506
ρ_{37}	0.381	0.420	-0.030	-0.061	0.932	0.136
ρ_{38}	0.485	0.351	-0.001	0.008	1.394	0.642
ρ_{39}	0.629	0.273	-0.008	-0.018	1.065	0.477
ρ_{40}	0.765	0.250	0.058	-0.009	0.808	0.353
ρ_{41}	0.760	0.192	0.012	0.001	0.826	0.268
ρ_{42}	0.192	0.387	-0.011	-0.032	1.404	0.249
ρ_{43}	0.715	0.235	0.043	-0.023	1.049	0.836
ρ_{44}	0.775	0.212	0.020	-0.019	1.171	0.430
ρ_{45}	0.762	0.278	0.005	0.041	1.111	0.221
ρ_{46}	0.276	0.436	0.051	-0.008	0.476	0.396
ρ_{47}	0.750	0.214	-0.004	-0.023	0.961	0.622
ρ_{48}	0.631	0.311	0.008	0.019	1.079	0.656
ρ_{49}	0.700	0.202	-0.003	0.040	1.008	0.756
ρ_{50}	0.544	0.334	0.001	0.008	1.712	0.671
ρ_{51}	0.677	0.288	-0.014	-0.013	0.856	0.242

Note: Statistics are based on 2,000 draws from the Gibbs sampler. Mean: average across draws; Sd.: standard deviation across draws; auto1: first order autocorrelation across draws; auto50: fiftieth order autocorrelation across draws; RNE: relative numerical efficiency using a 4 percent taper; p-value: p-value of the null hypothesis that the mean of the first 20 percent of draws is equal to the mean of the last 50 percent of draws using a 4 percent taper for the standard error.

Table M.9: Posterior Distribution Estimates and Convergence Diagnostics

Parameter	Mean	Sd.	auto1	auto50	RNE	p-value
$\sigma^2_{\zeta_1}$	0.170	0.051	0.029	0.032	0.886	0.182
$\sigma^2_{\zeta_2}$	0.667	0.758	0.134	0.009	1.104	0.403
$\sigma^2_{\zeta_3}$	0.249	0.094	0.085	-0.016	0.450	0.840
$\sigma^2_{\zeta_4}$	0.402	0.193	0.013	0.030	0.773	0.291
$\sigma^2_{\zeta_5}$	0.209	0.079	0.034	0.017	0.545	0.509
$\sigma^2_{\zeta_6}$	0.220	0.082	-0.004	0.031	0.909	0.964
$\sigma^2_{\zeta_7}$	0.266	0.132	0.037	0.000	0.616	0.079
$\sigma^2_{\zeta_8}$	0.413	0.585	0.285	-0.013	0.586	0.137
$\sigma^2_{\zeta_9}$	0.203	0.072	0.065	0.001	0.880	0.119
$\sigma^2_{\zeta_{10}}$	0.170	0.054	0.043	-0.045	0.662	0.988
$\sigma^2_{\zeta_{11}}$	0.149	0.046	-0.008	-0.013	0.994	0.384
$\sigma^2_{\zeta_{12}}$	0.154	0.044	-0.020	0.041	1.422	0.556
$\sigma^2_{\zeta_{13}}$	0.237	0.091	0.008	-0.003	0.605	0.312
$\sigma^2_{\zeta_{14}}$	0.156	0.045	0.010	0.033	1.223	0.809
$\sigma^2_{\zeta_{15}}$	0.220	0.079	-0.015	0.011	0.747	0.902
$\sigma^2_{\zeta_{16}}$	0.284	0.144	0.011	-0.006	0.824	0.280
$\sigma^2_{\zeta_{17}}$	0.428	0.251	-0.003	-0.038	1.026	0.250
$\sigma^2_{\zeta_{18}}$	0.163	0.056	-0.005	-0.035	0.667	0.563
$\sigma^2_{\zeta_{19}}$	0.394	0.180	0.029	0.002	0.994	0.495
$\sigma^2_{\zeta_{20}}$	0.179	0.061	0.023	0.029	1.063	0.640
$\sigma^2_{\zeta_{21}}$	0.158	0.050	0.016	-0.000	1.126	0.996
$\sigma^2_{\zeta_{22}}$	0.206	0.069	0.039	0.025	0.852	0.735
$\sigma^2_{\zeta_{23}}$	0.423	0.216	-0.011	0.000	0.681	0.779
$\sigma^2_{\zeta_{24}}$	0.244	0.092	-0.015	0.040	0.885	0.192
$\sigma^2_{\zeta_{25}}$	0.284	0.129	-0.032	-0.006	0.766	0.456
$\sigma^2_{\zeta_{26}}$	0.193	0.069	0.008	-0.007	0.965	0.030
$\sigma^2_{\zeta_{27}}$	0.255	0.112	-0.022	0.011	1.000	0.032
$\sigma^2_{\zeta_{28}}$	0.475	0.193	-0.025	-0.036	1.189	0.370
$\sigma^2_{\zeta_{29}}$	0.245	0.115	0.041	-0.040	0.542	0.139
$\sigma^2_{\zeta_{30}}$	0.305	0.128	0.023	-0.032	0.692	0.754
$\sigma^2_{\zeta_{31}}$	0.169	0.056	0.019	0.065	0.552	0.828
$\sigma^2_{\zeta_{32}}$	0.290	0.164	0.044	-0.033	0.893	0.676
$\sigma^2_{\zeta_{33}}$	0.269	0.174	0.021	0.003	1.016	0.503
$\sigma^2_{\zeta_{34}}$	0.276	0.123	0.035	0.011	0.915	0.407
$\sigma^2_{\zeta_{35}}$	0.496	0.658	0.170	0.007	1.032	0.831
$\sigma^2_{\zeta_{36}}$	0.210	0.076	-0.024	-0.002	0.750	0.001
$\sigma^2_{\zeta_{37}}$	0.580	0.574	0.040	-0.027	0.937	0.789
$\sigma^2_{\zeta_{38}}$	0.250	0.101	0.034	0.037	0.512	0.689
$\sigma^2_{\zeta_{39}}$	0.169	0.053	-0.027	0.003	0.855	0.261
$\sigma^2_{\zeta_{40}}$	0.350	0.158	0.031	0.014	1.078	0.931
$\sigma^2_{\zeta_{41}}$	0.154	0.045	-0.006	-0.008	1.103	0.984
$\sigma^2_{\zeta_{42}}$	0.829	0.729	0.099	0.001	0.882	0.403
$\sigma^2_{\zeta_{43}}$	0.174	0.058	0.021	0.003	1.041	0.727
$\sigma^2_{\zeta_{44}}$	0.332	0.152	0.025	0.030	0.795	0.943
$\sigma^2_{\zeta_{45}}$	0.236	0.093	0.003	0.013	1.086	0.833
$\sigma^2_{\zeta_{46}}$	0.194	0.078	0.018	-0.012	0.591	0.391
$\sigma^2_{\zeta_{47}}$	0.147	0.044	-0.006	-0.036	0.988	0.455
$\sigma^2_{\zeta_{48}}$	0.225	0.082	0.035	-0.013	0.843	0.252
$\sigma^2_{\zeta_{49}}$	0.333	0.149	0.010	-0.000	0.780	0.525
$\sigma^2_{\zeta_{50}}$	0.143	0.043	-0.030	0.037	0.895	0.083
$\sigma^2_{\zeta_{51}}$	0.600	0.807	0.242	0.012	0.536	0.732

Note: Statistics are based on 2,000 draws from the Gibbs sampler. Mean: average across draws; Sd.: standard deviation across draws; auto1: first order autocorrelation across draws; auto50: fiftieth order autocorrelation across draws; RNE: relative numerical efficiency using a 4 percent taper; p-value: p-value of the null hypothesis that the mean of the first 20 percent of draws is equal to the mean of the last 50 percent of draws using a 4 percent taper for the standard error.

Table M.10: Posterior Distribution Estimates and Convergence Diagnostics

Parameter	Mean	Sd.	auto1	auto50	RNE	p-value
$\sigma^2_{\eta^y_1}$	0.169	0.050	0.032	-0.030	0.915	0.066
$\sigma^2_{\eta^y_2}$	3.804	1.398	0.255	0.003	0.188	0.665
$\sigma^2_{\eta^y_3}$	0.285	0.098	0.039	0.029	0.807	0.432
$\sigma^2_{\eta^y_4}$	0.689	0.292	-0.005	0.015	0.967	0.281
$\sigma^2_{\eta^y_5}$	0.316	0.117	0.085	-0.007	0.403	0.367
$\sigma^2_{\eta^y_6}$	0.308	0.100	-0.012	0.005	1.653	0.328
$\sigma^2_{\eta^y_7}$	1.168	0.314	-0.002	0.011	0.874	0.480
$\sigma^2_{\eta^y_8}$	5.712	1.464	0.063	-0.051	1.041	0.656
$\sigma^2_{\eta^y_9}$	0.382	0.123	0.032	-0.018	1.332	0.004
$\sigma^2_{\eta^y_{10}}$	0.201	0.063	0.046	-0.004	0.396	0.279
$\sigma^2_{\eta^y_{11}}$	0.219	0.064	0.014	-0.031	1.136	0.499
$\sigma^2_{\eta^y_{12}}$	0.188	0.054	-0.002	0.041	1.143	0.607
$\sigma^2_{\eta^y_{13}}$	0.529	0.162	-0.008	0.006	1.144	0.561
$\sigma^2_{\eta^y_{14}}$	0.199	0.058	0.058	-0.004	1.046	0.907
$\sigma^2_{\eta^y_{15}}$	0.378	0.122	0.021	-0.028	0.752	0.812
$\sigma^2_{\eta^y_{16}}$	0.712	0.225	-0.028	-0.035	0.920	0.992
$\sigma^2_{\eta^y_{17}}$	1.342	0.453	-0.005	0.024	1.030	0.590
$\sigma^2_{\eta^y_{18}}$	0.263	0.086	0.045	0.015	0.562	0.349
$\sigma^2_{\eta^y_{19}}$	0.745	0.254	0.049	-0.006	0.752	0.183
$\sigma^2_{\eta^y_{20}}$	0.265	0.081	0.018	-0.030	0.663	0.704
$\sigma^2_{\eta^y_{21}}$	0.245	0.072	0.020	0.010	0.877	0.101
$\sigma^2_{\eta^y_{22}}$	0.249	0.083	-0.006	0.033	1.052	0.083
$\sigma^2_{\eta^y_{23}}$	0.765	0.270	0.015	-0.017	0.556	0.789
$\sigma^2_{\eta^y_{24}}$	0.395	0.130	-0.013	-0.019	1.808	0.527
$\sigma^2_{\eta^y_{25}}$	0.688	0.204	-0.017	0.029	1.405	0.796
$\sigma^2_{\eta^y_{26}}$	0.320	0.104	0.021	0.005	0.835	0.259
$\sigma^2_{\eta^y_{27}}$	0.521	0.161	0.015	0.008	0.995	0.270
$\sigma^2_{\eta^y_{28}}$	0.608	0.266	-0.016	-0.001	0.900	0.523
$\sigma^2_{\eta^y_{29}}$	0.485	0.154	0.026	-0.006	0.862	0.639
$\sigma^2_{\eta^y_{30}}$	0.547	0.203	-0.005	-0.006	1.076	0.767
$\sigma^2_{\eta^y_{31}}$	0.252	0.074	-0.012	-0.003	1.710	0.302
$\sigma^2_{\eta^y_{32}}$	0.980	0.318	0.133	-0.047	0.270	0.327
$\sigma^2_{\eta^y_{33}}$	1.756	0.474	0.029	-0.026	0.888	0.793
$\sigma^2_{\eta^y_{34}}$	0.646	0.202	0.009	-0.025	1.053	0.906
$\sigma^2_{\eta^y_{35}}$	4.445	1.167	0.045	0.023	1.696	0.468
$\sigma^2_{\eta^y_{36}}$	0.332	0.100	0.031	0.005	0.704	0.000
$\sigma^2_{\eta^y_{37}}$	3.031	0.989	0.058	-0.057	0.633	0.462
$\sigma^2_{\eta^y_{38}}$	0.547	0.172	0.027	-0.013	0.622	0.200
$\sigma^2_{\eta^y_{39}}$	0.232	0.071	-0.033	-0.033	0.847	0.487
$\sigma^2_{\eta^y_{40}}$	0.613	0.205	0.003	0.050	0.941	0.108
$\sigma^2_{\eta^y_{41}}$	0.173	0.052	0.007	0.005	0.997	0.076
$\sigma^2_{\eta^y_{42}}$	2.424	1.149	0.055	0.002	0.771	0.435
$\sigma^2_{\eta^y_{43}}$	0.286	0.087	-0.007	-0.016	0.741	0.244
$\sigma^2_{\eta^y_{44}}$	0.532	0.189	0.020	0.008	0.930	0.922
$\sigma^2_{\eta^y_{45}}$	0.396	0.134	0.037	-0.007	0.479	0.271
$\sigma^2_{\eta^y_{46}}$	0.301	0.107	0.049	-0.012	0.384	0.458
$\sigma^2_{\eta^y_{47}}$	0.175	0.050	-0.010	0.043	0.796	0.360
$\sigma^2_{\eta^y_{48}}$	0.364	0.113	-0.024	-0.000	0.984	0.860
$\sigma^2_{\eta^y_{49}}$	0.471	0.187	0.055	-0.002	0.457	0.968
$\sigma^2_{\eta^y_{50}}$	0.183	0.053	-0.008	-0.000	1.321	0.189
$\sigma^2_{\eta^y_{51}}$	5.062	1.443	0.230	-0.008	0.213	0.785

Note: Statistics are based on 2,000 draws from the Gibbs sampler. Mean: average across draws; Sd.: standard deviation across draws; auto1: first order autocorrelation across draws; auto50: fiftieth order autocorrelation across draws; RNE: relative numerical efficiency using a 4 percent taper; p-value: p-value of the null hypothesis that the mean of the first 20 percent of draws is equal to the mean of the last 50 percent of draws using a 4 percent taper

Table M.11: Posterior Distribution Estimates and Convergence Diagnostics

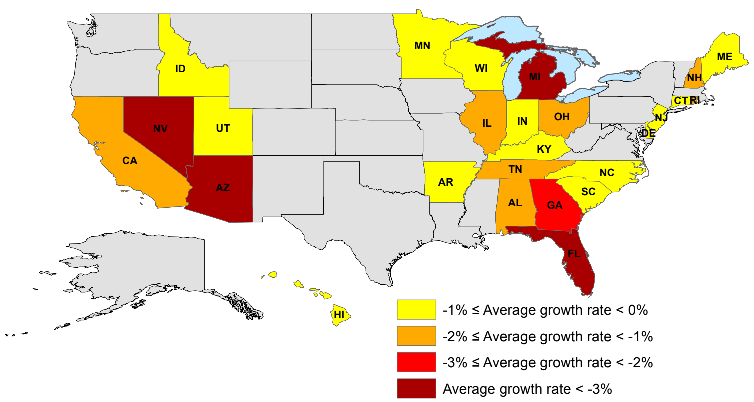
Parameter	Mean	Sd.	auto1	auto50	RNE	p-value
$\sigma^2_{\eta^u_1}$	0.086	0.018	0.000	-0.004	0.870	0.191
$\sigma^2_{\eta^u_2}$	0.048	0.010	-0.011	-0.048	1.116	0.178
$\sigma^2_{\eta^u_3}$	0.063	0.013	0.052	-0.029	0.847	0.007
$\sigma^2_{\eta^u_4}$	0.063	0.013	-0.027	-0.013	1.236	0.126
$\sigma^2_{\eta^u_5}$	0.060	0.013	0.000	-0.028	1.044	0.044
$\sigma^2_{\eta^u_6}$	0.070	0.015	-0.007	-0.012	1.975	0.778
$\sigma^2_{\eta^u_7}$	0.060	0.013	0.002	-0.024	1.821	0.292
$\sigma^2_{\eta^u_8}$	0.060	0.013	0.030	-0.031	0.870	0.641
$\sigma^2_{\eta^u_9}$	0.093	0.020	0.070	0.011	0.922	0.131
$\sigma^2_{\eta^u_{10}}$	0.061	0.013	0.006	0.014	0.950	0.192
$\sigma^2_{\eta^u_{11}}$	0.055	0.011	-0.002	0.015	0.740	0.911
$\sigma^2_{\eta^u_{12}}$	0.061	0.013	0.046	-0.009	0.845	0.148
$\sigma^2_{\eta^u_{13}}$	0.120	0.025	-0.011	0.028	1.126	0.186
$\sigma^2_{\eta^u_{14}}$	0.084	0.018	0.046	-0.014	0.942	0.388
$\sigma^2_{\eta^u_{15}}$	0.068	0.014	0.004	0.018	1.047	0.654
$\sigma^2_{\eta^u_{16}}$	0.049	0.010	0.013	-0.010	0.968	0.020
$\sigma^2_{\eta^u_{17}}$	0.049	0.010	0.014	-0.018	1.322	0.595
$\sigma^2_{\eta^u_{18}}$	0.072	0.015	0.032	0.011	1.091	0.640
$\sigma^2_{\eta^u_{19}}$	0.618	0.133	0.038	-0.044	1.026	0.308
$\sigma^2_{\eta^u_{20}}$	0.057	0.012	0.017	0.004	1.118	0.812
$\sigma^2_{\eta^u_{21}}$	0.053	0.011	0.005	-0.040	1.005	0.379
$\sigma^2_{\eta^u_{22}}$	0.058	0.012	0.014	-0.023	0.935	0.609
$\sigma^2_{\eta^u_{23}}$	0.089	0.020	0.016	0.017	1.047	0.204
$\sigma^2_{\eta^u_{24}}$	0.056	0.011	0.024	0.021	1.318	0.587
$\sigma^2_{\eta^u_{25}}$	0.142	0.032	0.016	-0.010	0.964	0.767
$\sigma^2_{\eta^u_{26}}$	0.067	0.014	-0.011	-0.008	1.277	0.775
$\sigma^2_{\eta^u_{27}}$	0.062	0.013	-0.007	0.054	0.680	0.655
$\sigma^2_{\eta^u_{28}}$	0.046	0.009	-0.019	-0.010	0.997	0.032
$\sigma^2_{\eta^u_{29}}$	0.087	0.019	0.002	-0.002	1.310	0.197
$\sigma^2_{\eta^u_{30}}$	0.058	0.012	-0.001	0.008	0.899	0.160
$\sigma^2_{\eta^u_{31}}$	0.072	0.015	-0.014	-0.012	0.902	0.196
$\sigma^2_{\eta^u_{32}}$	0.073	0.016	-0.009	-0.015	1.331	0.822
$\sigma^2_{\eta^u_{33}}$	0.061	0.013	0.033	0.011	0.626	0.821
$\sigma^2_{\eta^u_{34}}$	0.070	0.015	-0.012	-0.012	1.014	0.018
$\sigma^2_{\eta^u_{35}}$	0.048	0.010	-0.002	0.007	1.550	0.776
$\sigma^2_{\eta^u_{36}}$	0.068	0.015	-0.017	0.007	1.009	0.453
$\sigma^2_{\eta^u_{37}}$	0.061	0.013	0.005	0.061	1.165	0.177
$\sigma^2_{\eta^u_{38}}$	0.088	0.019	0.056	-0.009	0.587	0.688
$\sigma^2_{\eta^u_{39}}$	0.054	0.011	0.039	0.005	0.821	0.976
$\sigma^2_{\eta^u_{40}}$	0.081	0.019	-0.003	0.004	0.997	0.144
$\sigma^2_{\eta^u_{41}}$	0.073	0.016	0.003	-0.031	1.173	0.768
$\sigma^2_{\eta^u_{42}}$	0.053	0.011	0.029	0.001	1.180	0.274
$\sigma^2_{\eta^u_{43}}$	0.093	0.020	-0.004	-0.012	1.177	0.211
$\sigma^2_{\eta^u_{44}}$	0.064	0.013	-0.014	0.026	1.015	0.844
$\sigma^2_{\eta^u_{45}}$	0.069	0.015	0.019	0.026	1.424	0.557
$\sigma^2_{\eta^u_{46}}$	0.056	0.011	0.024	-0.019	1.396	0.524
$\sigma^2_{\eta^u_{47}}$	0.049	0.010	0.007	-0.024	1.063	0.594
$\sigma^2_{\eta^u_{48}}$	0.068	0.015	0.040	0.034	0.717	0.209
$\sigma^2_{\eta^u_{49}}$	0.091	0.020	0.032	0.041	0.898	0.325
$\sigma^2_{\eta^u_{50}}$	0.059	0.013	0.006	0.011	0.845	0.509
$\sigma^2_{\eta^u_{51}}$	0.080	0.018	-0.000	-0.006	1.034	0.323

Note: Statistics are based on 2,000 draws from the Gibbs sampler. Mean: average across draws; Sd.: standard deviation across draws; auto1: first order autocorrelation across draws; auto50: fiftieth order autocorrelation across draws; RNE: relative numerical efficiency using a 4 percent taper; p-value: p-value of the null hypothesis that the mean of the first 20 percent of draws is equal to the mean of the last 50 percent of draws using a 4 percent taper for the standard error.

Table M.12: Prior and Posterior Distributions of the Parameters using Aggregate Data

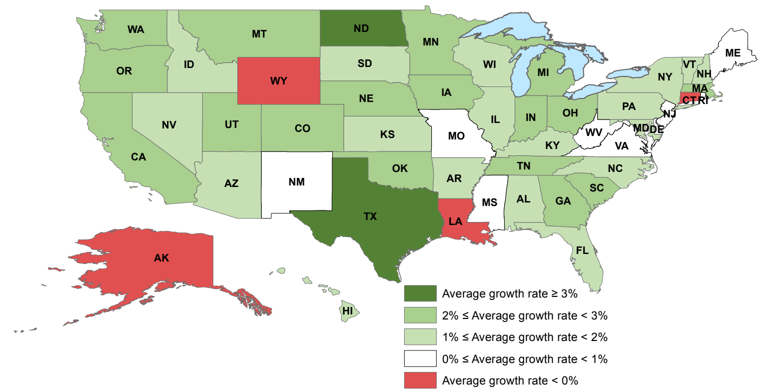
Parameter	Prior Distribution	Prior Mean	Prior Standard Deviation	Posterior Mean	Posterior Standard Deviation
μ	Normal	0.8	0.8	0.40	0.07
ϕ_1	Truncated Normal	1.5	0.5	1.55	0.13
ϕ_2	Truncated Normal	-0.6	0.2	-0.59	0.13
θ_1	Normal	-0.4	0.1	-0.39	0.07
θ_2	Normal	-0.1	0.1	-0.20	0.07
σ^2_{ε}	Inverse Gamma	1	Inf	0.21	0.06
$\sigma^2_{\eta^y}$	Inverse Gamma	1	Inf	0.18	0.05
$\sigma^2_{\eta^u}$	Inverse Gamma	1	Inf	0.07	0.02

Figure M.1: States with Negative Average Real GDP Annual Growth in the 2007–2009 Period



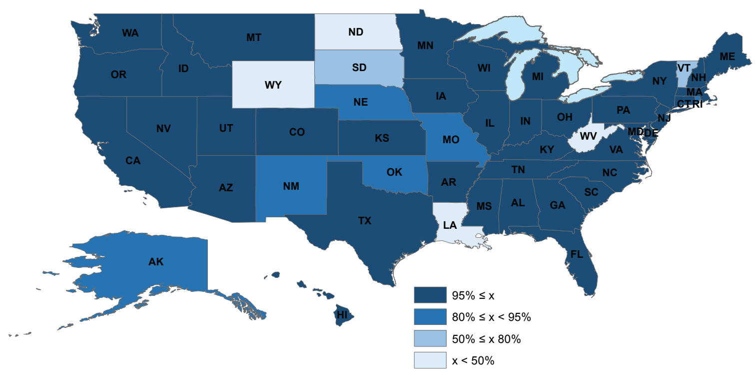
Note: States in light gray experienced positive average growth rates.
Source: BEA Regional Economic Accounts.

Figure M.2: Average Real GDP Annual Growth by State in the 2010–2016 Period



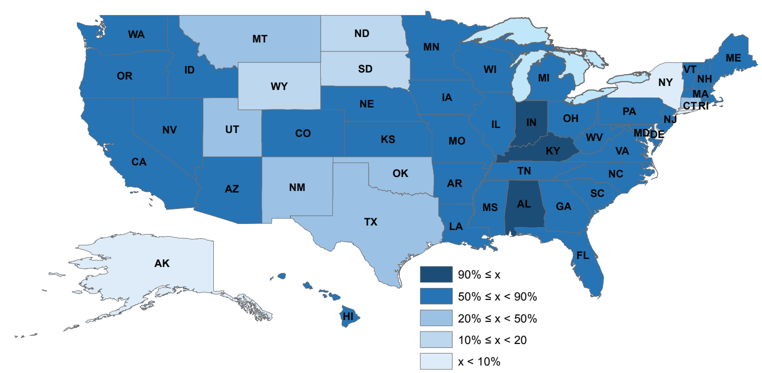
Source: BEA Regional Economic Accounts.

Figure M.3: Variance Decomposition of the State's Cycles



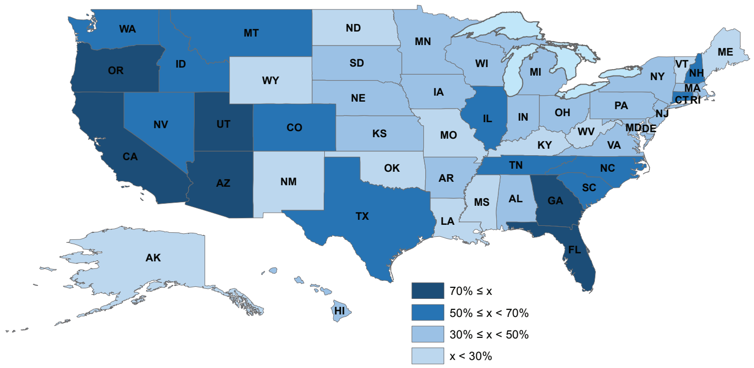
Note: $x = 100 \times \frac{\alpha_i^2 \, \mathrm{var}(c_t)}{\mathrm{var}(c_{it})}$ is the proportion of the variance of the state's cycle that is explained by the variance of the common cycle.

Figure M.4: Variance Decomposition of the State's Output Trends



Note: $x = 100 \times \frac{\left(\delta_i^y\right)^2}{\left(\delta_i^y\right)^2 + \sigma^2_{\eta_i^y}}$ is the proportion of the variance of the state's output trend that is explained by the variance of the common output trend.

Figure M.5: Variance Decomposition of the State's GDP Growth



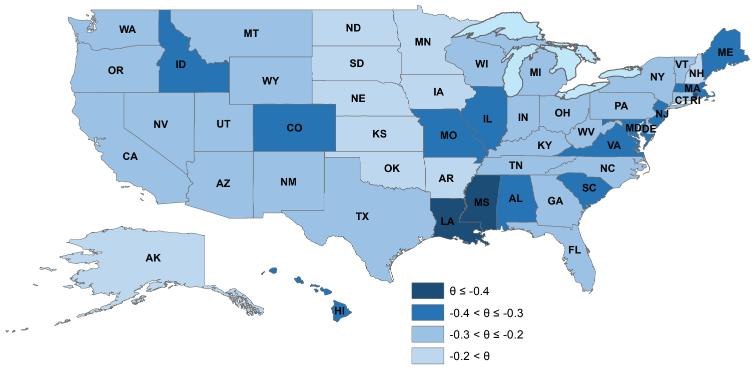
Note: $\mathrm{x} = 100 \times \frac{\alpha_i^2 \, \mathrm{var}(\Delta c_t)}{\mathrm{var}(\Delta c_{it})}$ is the proportion of the variance of the state's real GDP growth rate that is explained by the variance of its cycle.

Table M.13: Variance Decomposition of the State's GDP Growth Explained by Industry Composition, Demographics, and State Policy

Variable	Coefficient
Intercept	-1.52^{**}
Agriculture, forestry, fishing, and hunting	0.50
Mining, quarrying, and oil and gas extraction	-0.55
Construction	-1.64
Manufacturing	0.16
Wholesale trade; Retail trade; Transportation and warehousing	2.30^{*}
Finance and insurance; Real estate and rental and leasing	-0.04
Government and government enterprises	-1.23
Share of 25+ population with a bachelor's degree	-0.35
State's personal income tax as a share of personal income	-1.26
Share of population between the ages of 18 and 44	3.34^{***}
Debt-to-income ratio	0.85^{***}

Note: The dependent variable is the proportion of the variance of the state's real GDP growth rate that is explained by the variance of its cycle. Explanatory variables are expressed in their 2005–2017 averages. ***, **, and * denote statistical significance at the 1, 5, and 10 percent levels, respectively. $\bar{R}^2 = 0.55$.

Figure M.6: Okun's Law Coefficients ($\theta_{1i} + \theta_{2i}$)



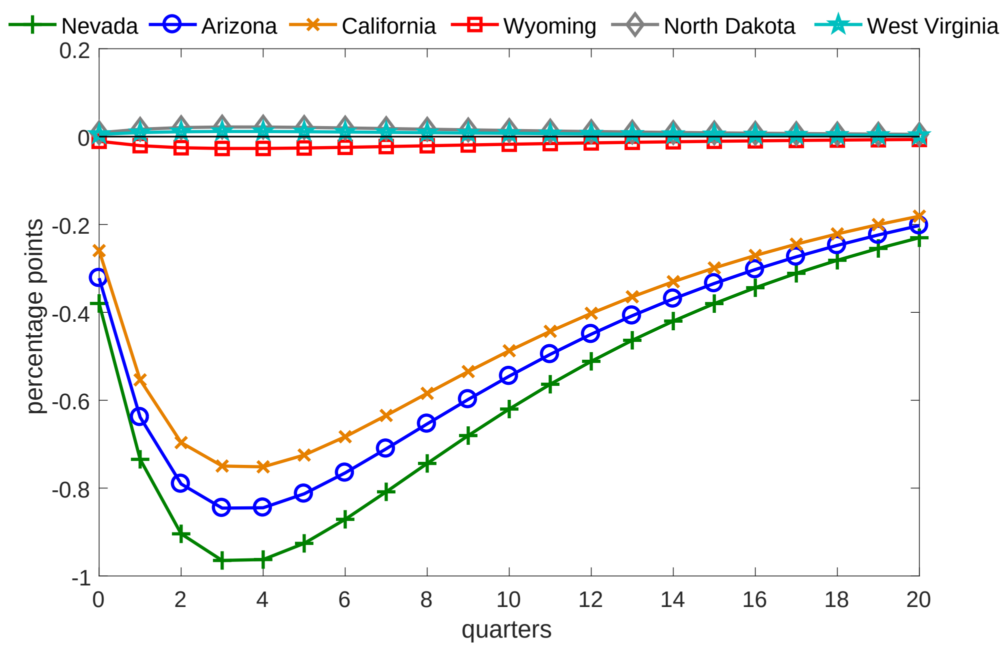
Note: θ is the sum $\theta_1 + \theta_2$ for each state. Source: Author's calculations.

Table M.14: State Labor Market Cyclicality Explained by Industry Composition, Demographics, State Policy, and Union Membership

Variable	Coefficient
Intercept	0.36
Agriculture, forestry, fishing, and hunting	−1.82***
Mining, quarrying, and oil and gas extraction	−0.46*
Construction	3.68***
Manufacturing	0.05
Wholesale trade; Retail trade; Transportation and warehousing	−1.07
Finance and insurance; Real estate and rental and leasing	−0.23
Government and government enterprises	0.24
Share of 25+ population with a bachelor's degree	−0.45
State's personal income tax as a share of personal income	0.70
Share of population between the ages of 18 and 44	0.10
Union membership	0.00

Note: The dependent variable is the absolute value of the sum $\theta_{1i} + \theta_{2i}$ for each state. Explanatory variables are expressed in their 2005–2017 averages. ***, **, and * denote statistical significance at the 1, 5, and 10 percent levels, respectively. $\bar{R} = 0.40$.

Figure M.7: Impulse-Response Analysis for the Cyclical Component of the Unemployment Rate



Note: Response to a 0.8 percentage point increase in the aggregate output gap.