Supplementary material to Forecasting with the Standardized Self-Perturbed Kalman Filter

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A Robustness to the initial conditions

The setups considered for the initial conditions of H_0, θ_0 and P_0 are:

(I) A diffuse prior centered around the null-hypothesis of no-predictability with the starting value of the H as the square of the first observation: $\mathrm{H}_0 = y_1^2$, $\theta_0 = 0$ and $\mathrm{P}_0 = 100 \times \mathrm{I}$.

(II) A diffuse prior centered around the null-hypothesis of no-predictability with the starting value of the H as the variance of the first 30 observations (training sample): $\mathrm{H}_0 = \mathrm{Var}(y_1, \dots, y_{30})$, $\theta_0 = 0$ and $\mathrm{P}_0 = 100 \times \mathrm{I}$.

(III) A diffuse prior centered around the null-hypothesis of no-predictability with the starting value of the H as the variance of the first 50 observations (training sample): $\mathrm{H}_0 = \mathrm{Var}(y_1, \dots, y_{50})$, $\theta_0 = 0$ and $\mathrm{P}_0 = 100 \times \mathrm{I}$.

(IV) An OLS initialization of H_0, θ_0 and P_0 regressing $y_{1,\dots,30}$ on $X_{i,1,\dots,30}, i = 1, 2$ with a training sample of 30 observations.

(V) An OLS initialization of H_0, θ_0 and P_0 regressing $y_{1,\dots,50}$ on $X_{i,1,\dots,50}, i = 1, 2$ with a training sample of 50 observations.

The last two initialization are similar to Dangl and Halling (2012), we also tried to use a $g - prior$ with different scale but the results do not change. The following table reports the results for the 5 different choices of the priors.

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	I	II	III	IV	V
Gaussian:					
0B	1.0000	0.9985	0.9984	0.9983	0.9973
1B	1.0000	1.0025	1.0025	1.0043	1.0073
2B	1.0000	1.0033	1.0035	1.0035	1.0054
3B	1.0000	1.0020	1.0022	1.0021	1.0046
RW	1.0000	1.0017	1.0021	1.0015	1.0059
GARCH:					
0B	1.0000	0.9941	0.9941	0.9941	0.9937
1B	1.0000	1.0050	1.0051	1.0091	1.0115
2B	1.0000	1.0016	1.0019	1.0035	1.0041
3B	1.0000	1.0039	1.0040	1.0049	1.0069
RW	1.0000	1.0005	1.0007	1.0000	1.0053
Outliers:					
0B	1.0000	0.9995	0.9991	0.9992	0.9985
1B	1.0000	1.0025	1.0023	1.0030	1.0035
2B	1.0000	1.0006	0.9993	0.9995	0.9997
3B	1.0000	1.0018	1.0010	1.0012	1.0020
RW	1.0000	1.0019	1.0021	1.0013	1.0031

Table 1: Robustness to alternative initialization schemes. Table reports the Monte Carlo average of the absolute parameter deviations (APD) obtained with the $SSP_{\varsigma,\kappa}$ estimator based on different initialization schemes. We consider 5 initialization schemes described before. The reported values are normalized with respect to the APD of the first initialization scheme. Hence the first column contains only ones. The reported numbers are based on the $SSP_{\varsigma,\kappa}$ estimates carried out on 1000 Monte Carlo replications of models with the following parameter dynamics: no breaks (0B), 1 break (1B), 2 breaks (2B), three breaks (3B) and random walk (RW). The sample size is $T = 500$. Three types of innovations are considered: iid Gaussian, GARCH errors and outliers from a Student's t with 3 degrees of freedom. The level of noise-to-signal ratio is 1.

B Other Monte Carlo results

Table 2: Monte Carlo. Table reports the 1-step ahead absolute parameter distance relative to that of the Kalman Filter of several estimators of TVP models. The considered estimators are the following: 1) OLS; 2) forgetting factor with constant parameters (CFF); 3) Forgetting factor with the dynamic selection of λ and κ (KK), with $\lambda \in [0.9, 0.91, ..., 0.99]$ and $\kappa \in [0.94, 0.96, 0.98]$ as in Koop and Korobilis (2013); 4) the self-perturbed Kalman filter of Park (1992) (SP) with dynamic selection of $\varsigma, \kappa, \gamma$ with $\varsigma \in [0.01, 0.02, 0.03, 0.04]$, $\kappa \in [0.94, 0.96, 0.98]$ and $\gamma \in [0.01, 0.21, 0.41, 0.61, 0.81, 1.01, 1.21, 1.41]$; 5) the standardized self-perturbed Kalman filter, (SSP), with dynamic selection of ς, κ with $\varsigma \in [0.01, 0.02, 0.03, 0.04]$ and $\kappa \in [0.94, 0.96, 0.98]$; 6) MCMC with Kalman Filter for TVP model (KF-MCMC); 7) MCMC with Kalman Filter for TVP model under stochastic volatility (KF-MCMC-SV); 8) Change Point model of Pesaran et al (2006) with different number of breaks percentages. The dynamic selection of the design parameters λ, ς, κ and γ has been performed with DMS for different values of $\alpha \in [0.001, 0.95, 1]$. The sample size is T=250.

	No Breaks					**One Break**					**Three Breaks**					**Random Walk**				
	0.1	**0.5**	**1**	**5**	**10**	**0.1**	**0.5**	**1**	**5**	**10**	**0.1**	**0.5**	**1**	**5**	**10**	**0.1**	**0.5**	**1**	**5**	**10**
iid Gaussian:																				
OLS	0.70	0.70	0.70	0.70	0.70	4.30	2.76	2.27	1.41	1.15	3.59	2.31	1.89	1.23	1.05	1.71	1.18	1.04	0.85	0.81
CFF-$\lambda = 0.96$, $\kappa = 0.94$	1.61	1.57	1.55	1.56	1.54	2.34	1.28	1.09	0.96	0.98	2.96	1.62	1.31	0.97	0.95	1.20	1.03	1.08	1.24	1.35
CFF-$\lambda = 0.98$, $\kappa = 0.94$	1.27	1.26	1.21	1.22	1.20	4.52	2.04	1.57	1.06	0.96	3.99	2.25	1.79	1.15	1.01	1.49	1.16	1.13	1.07	1.10
KK, $\alpha = 0.001$	1.16	1.18	1.16	1.15	1.14	4.17	2.06	1.63	1.15	1.03	3.84	2.25	1.81	1.21	1.07	1.48	1.21	1.18	1.06	1.06
KK, $\alpha = 0.95$	1.05	1.06	1.05	1.06	1.05	4.07	1.93	1.51	1.16	1.08	3.68	2.09	1.67	1.18	1.07	1.45	1.19	1.17	1.06	1.04
KK, $\alpha = 1$	1.05	1.08	1.07	1.09	1.08	4.38	2.00	1.57	1.20	1.13	3.75	2.10	1.68	1.17	1.07	1.46	1.20	1.19	1.07	1.05
SP$_{\varsigma,\kappa,\gamma}$, $\alpha = 0.001$	1.08	1.61	1.89	2.47	2.72	1.09	0.97	1.03	1.33	1.60	1.39	1.05	1.05	1.23	1.47	1.68	1.17	1.26	1.99	2.41
SP$_{\varsigma,\kappa,\gamma}$, $\alpha = 0.95$	1.05	1.35	1.33	1.25	1.21	0.98	0.99	1.01	1.05	1.04	1.27	1.10	1.10	1.07	1.03	1.70	1.13	1.09	1.11	1.10
SP$_{\varsigma,\kappa,\gamma}$, $\alpha = 1$	1.05	1.28	1.25	1.21	1.20	0.98	1.05	1.07	1.11	1.07	1.27	1.14	1.13	1.12	1.05	1.72	1.14	1.12	1.08	1.07
SSP$_{\varsigma,\kappa}$, $\alpha = 0.001$	2.92	2.37	2.20	1.96	1.92	0.95	1.03	1.06	1.13	1.18	1.07	1.08	1.08	1.08	1.11	1.19	1.23	1.28	1.55	1.65
SSP$_{\varsigma,\kappa}$, $\alpha = 0.95$	1.44	1.30	1.31	1.26	1.25	0.97	1.00	1.03	1.07	1.08	1.11	1.09	1.09	1.06	1.04	1.13	1.08	1.08	1.11	1.13
SSP$_{\varsigma,\kappa}$, $\alpha = 1$	1.40	1.25	1.24	1.23	1.23	1.24	1.12	1.10	1.16	1.13	1.25	1.18	1.14	1.09	1.08	1.19	1.13	1.11	1.08	1.09
KF-MCMC	2.76	2.13	1.92	1.57	1.46	0.81	0.83	0.83	0.85	0.87	0.78	0.76	0.75	0.76	0.79	0.86	0.95	1.01	1.18	1.21
KF-MCMC-SV	3.25	2.48	2.22	1.77	1.61	0.94	0.94	0.95	0.97	0.97	0.83	0.81	0.81	0.87	0.88	1.10	1.16	1.19	1.29	1.29
ChagePoint 0.2%	1.57	1.53	1.38	1.51	1.13	1.66	1.15	1.01	0.93	1.03	3.15	2.03	1.66	1.20	1.12	1.36	1.24	1.12	1.20	1.32
ChagePoint 2%	5.65	3.09	2.67	2.13	1.91	0.99	1.09	1.12	1.21	1.33	1.01	1.06	1.17	1.32	1.35	1.88	1.94	1.69	1.85	1.90
ChagePoint 10%	8.75	6.87	5.85	4.05	3.59	1.87	1.96	1.94	2.06	1.85	1.35	1.71	1.68	1.63	1.61	3.12	3.02	2.93	3.08	2.93
Student's t(3):																				
OLS	0.65	0.65	0.65	0.65	0.65	3.37	2.10	1.69	1.01	0.85	2.82	1.75	1.43	0.96	0.85	1.34	0.97	0.89	0.76	0.73
CFF-$\lambda = 0.96$, $\kappa = 0.94$	1.29	1.21	1.20	1.18	1.16	1.58	1.02	0.91	0.84	0.89	2.12	1.21	1.02	0.86	0.89	1.00	0.97	1.00	1.17	1.20
CFF-$\lambda = 0.98$, $\kappa = 0.94$	1.12	0.98	0.96	0.94	0.90	2.88	1.49	1.19	0.84	0.80	2.93	1.66	1.33	0.92	0.85	1.21	1.05	0.96	0.97	0.93
KK, $\alpha = 0.001$	1.05	0.97	0.95	0.91	0.88	2.79	1.54	1.28	0.94	0.87	2.87	1.68	1.39	0.99	0.90	1.25	1.12	1.03	0.98	0.90
KK, $\alpha = 0.95$	0.94	0.85	0.85	0.81	0.78	2.70	1.46	1.21	0.95	0.88	2.73	1.58	1.31	0.98	0.90	1.21	1.08	1.00	0.93	0.84
KK, $\alpha = 1$	0.95	0.87	0.87	0.83	0.79	2.88	1.51	1.26	0.98	0.90	2.78	1.58	1.31	0.99	0.90	1.23	1.10	1.02	0.94	0.85
SP$_{\varsigma,\kappa,\gamma}$, $\alpha = 0.001$	1.21	1.43	1.58	1.92	2.07	0.96	0.95	1.00	1.28	1.54	1.12	0.98	0.97	1.23	1.49	1.12	1.11	1.29	1.93	2.20
SP$_{\varsigma,\kappa,\gamma}$, $\alpha = 0.95$	1.09	1.03	0.99	0.96	0.94	0.96	0.94	0.93	0.89	0.89	1.09	1.02	0.98	0.93	0.91	1.12	0.95	0.98	0.97	0.94
SP$_{\varsigma,\kappa,\gamma}$, $\alpha = 1$	1.05	0.99	0.96	0.94	0.91	1.03	1.02	1.01	0.90	0.88	1.15	1.07	1.04	0.93	0.90	1.15	0.98	0.98	0.94	0.91
SSP$_{\varsigma,\kappa}$, $\alpha = 0.001$	1.85	1.61	1.52	1.47	1.43	0.93	1.00	1.01	1.05	1.12	1.08	1.06	1.03	1.04	1.10	1.08	1.13	1.22	1.36	1.41
SSP$_{\varsigma,\kappa}$, $\alpha = 0.95$	1.06	1.00	0.99	0.97	0.95	0.95	0.96	0.97	0.92	0.90	1.08	1.02	0.99	0.94	0.93	1.01	0.97	0.98	0.95	0.94
SSP$_{\varsigma,\kappa}$, $\alpha = 1$	0.98	0.94	0.94	0.91	0.88	1.17	1.05	1.07	0.98	0.90	1.21	1.08	1.05	0.99	0.93	1.08	1.01	0.98	0.92	0.89
KF-MCMC	2.06	1.64	1.51	1.27	1.20	0.82	0.83	0.83	0.84	0.88	0.78	0.75	0.75	0.79	0.84	0.89	1.02	1.10	1.19	1.18
KF-MCMC-SV	2.12	1.62	1.45	1.13	1.01	0.87	0.84	0.82	0.78	0.77	0.76	0.73	0.73	0.74	0.74	1.01	1.08	1.11	1.07	1.01
ChagePoint 0.2%	1.10	1.24	0.97	1.02	0.91	1.37	1.01	0.99	0.98	0.90	2.45	1.56	1.34	1.08	1.02	1.23	1.03	1.03	1.28	1.17
ChagePoint 2%	2.97	2.15	1.87	1.93	1.67	0.82	1.17	1.44	1.45	1.42	0.98	1.27	1.30	1.40	1.29	1.88	1.57	2.15	1.69	1.96
ChagePoint 10%	5.49	3.91	4.04	3.86	3.44	1.57	2.08	2.17	2.29	2.23	1.47	1.71	1.82	2.01	1.87	2.37	2.96	3.11	3.47	3.18
GARCH(1,1):																				
OLS	0.66	0.66	0.66	0.66	0.66	4.31	2.73	2.23	1.33	1.06	3.56	2.27	1.85	1.17	1.01	1.48	1.05	0.93	0.80	0.72
CFF-$\lambda = 0.96$, $\kappa = 0.94$	1.54	1.51	1.53	1.47	1.46	2.43	1.28	1.12	0.96	0.95	2.95	1.65	1.32	0.97	0.94	1.13	1.01	1.06	1.31	1.32
CFF-$\lambda = 0.98$, $\kappa = 0.94$	1.26	1.24	1.29	1.16	1.14	4.65	2.08	1.60	1.06	0.95	3.96	2.25	1.77	1.12	0.99	1.32	1.04	1.02	1.11	1.04
KK, $\alpha = 0.001$	1.16	1.15	1.19	1.07	1.05	4.37	2.11	1.68	1.16	1.03	3.83	2.25	1.80	1.18	1.04	1.32	1.08	1.04	1.05	0.98
KK, $\alpha = 0.95$	1.05	1.03	1.08	1.01	0.99	4.35	2.07	1.64	1.18	1.07	3.68	2.11	1.68	1.16	1.05	1.31	1.08	1.05	1.02	0.91
KK, $\alpha = 1$	1.05	1.03	1.08	1.02	1.01	4.76	2.16	1.71	1.25	1.13	3.78	2.15	1.71	1.17	1.07	1.34	1.12	1.08	1.00	0.90
SP$_{\varsigma,\kappa,\gamma}$, $\alpha = 0.001$	1.09	1.59	1.89	2.36	2.58	1.07	0.99	1.04	1.33	1.55	1.40	1.08	1.05	1.19	1.43	1.53	1.22	1.38	2.21	2.53
SP$_{\varsigma,\kappa,\gamma}$, $\alpha = 0.95$	1.06	1.35	1.38	1.23	1.18	0.98	1.01	1.03	1.05	1.02	1.29	1.10	1.09	1.04	1.01	1.53	1.12	1.12	1.29	1.22
SP$_{\varsigma,\kappa,\gamma}$, $\alpha = 1$	1.06	1.28	1.28	1.11	1.08	0.99	1.09	1.11	1.13	1.06	1.28	1.15	1.14	1.09	1.03	1.57	1.13	1.03	1.05	0.98
SSP$_{\varsigma,\kappa}$, $\alpha = 0.001$	2.76	2.33	2.17	1.95	1.90	0.98	1.05	1.09	1.15	1.20	1.09	1.11	1.10	1.08	1.13	1.20	1.31	1.43	1.76	1.82
SSP$_{\varsigma,\kappa}$, $\alpha = 0.95$	1.47	1.28	1.25	1.22	1.20	0.99	1.04	1.06	1.07	1.05	1.13	1.11	1.09	1.03	1.02	1.13	1.10	1.14	1.28	1.23
SSP$_{\varsigma,\kappa}$, $\alpha = 1$	1.40	1.20	1.16	1.12	1.10	1.28	1.17	1.16	1.16	1.10	1.27	1.20	1.16	1.08	1.04	1.16	1.05	1.04	1.04	1.02
KF-MCMC	2.95	2.22	2.00	1.57	1.42	0.93	0.91	0.91	0.90	0.89	0.83	0.80	0.80	0.82	0.82	1.07	1.15	1.20	1.36	1.32
KF-MCMC-SV	2.51	1.95	1.77	1.46	1.37	0.80	0.80	0.81	0.81	0.82	0.77	0.75	0.75	0.73	0.77	0.93	1.07	1.15	1.37	1.35
ChagePoint 0.2%	2.03	1.42	1.08	1.08	0.95	1.68	1.16	1.01	0.91	0.93	3.16	1.99	1.61	1.14	1.02	1.26	1.35	3.31	1.69	0.93
ChagePoint 2%	4.29	2.27	2.14	1.98	1.85	0.90	0.95	1.10	1.34	1.31	1.00	1.05	1.12	1.22	1.21	2.00	1.46	1.49	1.98	2.03
ChagePoint 10%	7.56	5.44	5.64	3.72	3.44	1.99	1.96	2.00	1.94	1.86	1.45	1.66	1.63	1.58	1.59	2.49	2.94	3.09	3.00	2.87

Table 3: Monte Carlo. Table reports the 1-step ahead absolute parameter distance relative to that of the Kalman Filter of several estimators of TVP models. The considered estimators are the following: 1) OLS; 2) forgetting factor with constant parameters (CFF); 3) Forgetting factor with the dynamic selection of λ and κ (KK), with $\lambda \in [0.9, 0.91, ..., 0.99]$ and $\kappa \in [0.94, 0.96, 0.98]$ as in Koop and Korobilis (2013); 4) the self-perturbed Kalman filter of Park (1992) (SP) with dynamic selection of $\varsigma, \kappa, \gamma$ with $\varsigma \in [0.01, 0.02, 0.03, 0.04]$, $\kappa \in [0.94, 0.96, 0.98]$ and $\gamma \in [0.01, 0.21, 0.41, 0.61, 0.81, 1.01, 1.21, 1.41]$; 5) the standardized self-perturbed Kalman filter, (SSP), with dynamic selection of ς, κ with $\varsigma \in [0.01, 0.02, 0.03, 0.04]$ and $\kappa \in [0.94, 0.96, 0.98]$; 6) MCMC with Kalman Filter for TVP model (KF-MCMC); 7) MCMC with Kalman Filter for TVP model under stochastic volatility (KF-MCMC-SV); 8) Change Point model of Pesaran et al (2006) with different number of breaks percentages. The dynamic selection of the design parameters λ, ς, κ and γ has been performed with DMS for different values of $\alpha \in [0.001, 0.95, 1]$. The sample size is T=1000.

	No Breaks					**One Break**					**Three Breaks**					**Random Walk**				
	0.1	**0.5**	**1**	**5**	**10**	**0.1**	**0.5**	**1**	**5**	**10**	**0.1**	**0.5**	**1**	**5**	**10**	**0.1**	**0.5**	**1**	**5**	**10**
iid Gaussian:																				
OLS	0.56	0.56	0.56	0.56	0.56	7.09	4.55	3.73	2.29	1.82	5.70	3.68	3.04	1.90	1.54	3.34	2.26	1.91	1.29	1.09
CFF-$\lambda = 0.96$, $\kappa = 0.94$	2.42	2.43	2.39	2.40	2.39	1.28	1.03	1.06	1.26	1.37	1.45	1.05	1.02	1.10	1.20	1.19	1.05	1.09	1.38	1.55
CFF-$\lambda = 0.98$, $\kappa = 0.94$	1.77	1.81	1.73	1.75	1.73	1.82	1.14	1.05	1.03	1.07	2.43	1.41	1.21	1.00	0.99	1.55	1.15	1.07	1.10	1.18
KK, $\alpha = 0.001$, SG	1.42	1.47	1.41	1.44	1.41	1.80	1.18	1.09	1.01	1.00	2.32	1.44	1.28	1.08	1.03	1.57	1.22	1.13	1.05	1.07
KK, $\alpha = 0.95$, SG	1.12	1.18	1.14	1.18	1.14	1.81	1.18	1.09	1.03	1.02	2.14	1.31	1.17	1.07	1.06	1.45	1.18	1.12	1.04	1.03
KK, $\alpha = 1$, SG	1.03	1.13	1.14	1.20	1.15	2.12	1.33	1.23	1.18	1.17	2.31	1.33	1.19	1.16	1.17	1.37	1.16	1.14	1.14	1.12
$SP_{\varsigma,\kappa,\gamma}$, $\alpha = 0.001$	0.99	1.87	2.25	3.06	3.45	0.78	0.90	1.03	1.61	2.00	1.04	0.97	1.04	1.47	1.83	1.79	1.19	1.24	1.93	2.43
$SP_{\varsigma,\kappa,\gamma}$, $\alpha = 0.95$	0.97	1.46	1.52	1.53	1.50	0.74	0.85	0.91	1.05	1.08	0.95	0.93	0.98	1.05	1.07	1.81	1.14	1.09	1.11	1.15
$SP_{\varsigma,\kappa,\gamma}$, $\alpha = 1$	0.97	1.21	1.20	1.19	1.15	0.70	0.96	1.03	1.12	1.09	0.92	1.02	1.06	1.16	1.11	1.77	1.14	1.09	1.09	1.09
$SSP_{\varsigma,\kappa}$, $\alpha = 0.001$	3.69	2.91	2.68	2.40	2.36	0.98	1.05	1.10	1.27	1.37	0.95	1.02	1.06	1.14	1.21	1.13	1.16	1.19	1.41	1.55
$SSP_{\varsigma,\kappa}$, $\alpha = 0.95$	1.72	1.59	1.57	1.52	1.49	0.81	0.91	0.97	1.07	1.11	0.87	0.96	0.99	1.05	1.07	1.08	1.07	1.08	1.12	1.16
$SSP_{\varsigma,\kappa}$, $\alpha = 1$	1.45	1.24	1.22	1.20	1.17	1.03	1.11	1.10	1.17	1.22	1.08	1.07	1.07	1.10	1.17	1.08	1.09	1.10	1.12	1.09
KF-MCMC	3.29	2.55	2.32	1.95	1.83	0.84	0.88	0.91	1.00	1.04	0.78	0.80	0.81	0.86	0.90	0.79	0.86	0.91	1.08	1.17
KF-MCMC-SV	4.10	3.17	2.86	2.29	2.08	1.01	1.05	1.08	1.13	1.14	0.88	0.91	0.92	0.95	0.96	0.94	1.02	1.07	1.20	1.25
ChagePoint 0.2%	2.79	1.45	1.84	0.88	1.07	0.78	0.63	0.62	0.79	0.96	2.63	1.79	1.46	1.16	1.21	1.77	1.40	1.24	1.21	1.06
ChagePoint 2%	5.87	4.68	3.83	3.22	3.12	1.55	1.69	1.81	1.99	1.85	1.19	1.50	1.66	1.85	1.80	1.72	2.03	2.12	2.06	1.91
ChagePoint 10%	10.32	8.06	7.77	6.25	5.53	2.93	2.99	2.83	2.79	2.85	2.32	2.43	2.55	2.41	2.24	2.93	3.11	3.17	3.16	2.84
Student's t(3):																				
OLS	0.54	0.54	0.54	0.54	0.54	5.44	3.35	2.69	1.57	1.23	4.41	2.74	2.21	1.35	1.09	2.60	1.75	1.47	0.98	0.84
CFF-$\lambda = 0.96$, $\kappa = 0.94$	1.99	1.99	1.98	1.98	1.96	1.01	0.95	1.00	1.17	1.27	1.12	0.92	0.92	1.04	1.14	1.00	0.99	1.06	1.35	1.51
CFF-$\lambda = 0.98$, $\kappa = 0.94$	1.46	1.47	1.45	1.42	1.40	1.32	0.94	0.90	0.91	0.95	1.72	1.09	0.97	0.86	0.88	1.23	0.98	0.94	1.02	1.10
KK, $\alpha = 0.001$, SG	1.17	1.22	1.20	1.17	1.15	1.37	1.01	0.96	0.89	0.89	1.70	1.17	1.07	0.92	0.88	1.28	1.05	0.99	0.94	0.96
KK, $\alpha = 0.95$, SG	0.90	0.95	0.94	0.91	0.89	1.37	0.99	0.94	0.89	0.87	1.57	1.06	1.00	0.93	0.89	1.21	1.04	0.99	0.90	0.87
KK, $\alpha = 1$, SG	0.83	0.94	0.95	0.91	0.87	1.55	1.12	1.08	1.01	1.00	1.63	1.09	1.06	1.02	0.96	1.16	1.07	1.05	1.00	0.95
$SP_{\varsigma,\kappa,\gamma}$, $\alpha = 0.001$	1.43	1.86	2.10	2.74	3.01	0.80	0.94	1.07	1.65	2.00	0.90	0.94	1.03	1.52	1.86	1.18	1.11	1.26	2.03	2.49
$SP_{\varsigma,\kappa,\gamma}$, $\alpha = 0.95$	1.21	1.23	1.24	1.22	1.20	0.77	0.83	0.87	0.94	0.94	0.87	0.89	0.90	0.93	0.93	1.18	0.98	0.97	0.99	1.03
$SP_{\varsigma,\kappa,\gamma}$, $\alpha = 1$	1.02	0.97	0.96	0.93	0.91	0.89	0.95	0.96	0.94	0.93	0.96	0.97	0.98	0.97	0.93	1.15	0.99	0.99	0.95	0.91
$SSP_{\varsigma,\kappa}$, $\alpha = 0.001$	2.55	2.17	2.08	1.99	1.97	0.92	1.01	1.07	1.22	1.31	0.92	0.97	0.99	1.10	1.18	1.06	1.10	1.16	1.39	1.53
$SSP_{\varsigma,\kappa}$, $\alpha = 0.95$	1.34	1.28	1.25	1.21	1.20	0.80	0.87	0.91	0.96	0.96	0.87	0.90	0.91	0.94	0.93	0.99	0.98	0.98	1.00	1.03
$SSP_{\varsigma,\kappa}$, $\alpha = 1$	1.08	1.03	1.02	0.94	0.91	1.01	1.00	1.01	1.01	0.89	1.03	0.96	0.95	1.09	0.97	1.01	1.03	1.03	0.92	0.87
KF-MCMC	2.70	2.21	2.05	1.80	1.71	0.87	0.93	0.96	1.05	1.10	0.80	0.82	0.83	0.91	0.97	0.81	0.92	0.99	1.19	1.30
KF-MCMC-SV	2.84	2.21	1.99	1.59	1.44	0.91	0.91	0.91	0.91	0.90	0.79	0.78	0.77	0.78	0.78	0.84	0.91	0.94	1.01	1.03
ChagePoint 0.2%	2.11	1.64	0.95	1.44	1.65	0.75	0.97	1.30	1.34	1.27	1.57	1.543	1.28	1.51	1.17	2.30	1.66	1.53	1.38	1.24
ChagePoint 2%	4.28	3.67	3.46	3.58	3.41	1.22	1.53	1.61	2.12	2.00	1.20	1.57	1.76	2.07	1.88	1.08	1.29	1.37	1.66	1.59
ChagePoint 10%	7.82	7.29	7.26	6.50	5.60	2.37	2.60	2.96	3.17	3.21	2.29	2.84	2.71	3.07	2.58	1.98	2.28	2.43	2.62	2.30
GARCH(1,1):																				
OLS	0.62	0.62	0.62	0.63	0.63	7.17	4.55	3.72	2.29	1.83	5.72	3.65	2.99	1.87	1.52	2.36	1.57	1.34	0.96	0.85
CFF-$\lambda = 0.96$, $\kappa = 0.94$	2.65	2.65	2.65	2.65	2.64	1.31	1.03	1.05	1.25	1.37	1.48	1.06	1.01	1.10	1.19	1.12	1.03	1.09	1.46	1.68
CFF-$\lambda = 0.98$, $\kappa = 0.94$	1.90	1.92	1.95	1.89	1.88	1.84	1.15	1.03	1.01	1.06	2.48	1.41	1.20	1.00	0.99	1.42	1.05	1.00	1.13	1.25
KK, $\alpha = 0.001$, SG	1.51	1.54	1.59	1.52	1.50	1.81	1.19	1.07	1.00	0.99	2.37	1.45	1.27	1.07	1.02	1.45	1.11	1.02	1.02	1.08
KK, $\alpha = 0.95$, SG	1.17	1.21	1.26	1.19	1.18	1.84	1.20	1.07	1.01	1.01	2.19	1.32	1.16	1.06	1.04	1.34	1.07	1.02	0.96	0.96
KK, $\alpha = 1$, SG	1.04	1.15	1.23	1.13	1.11	2.26	1.35	1.21	1.17	1.16	2.37	1.35	1.18	1.13	1.13	1.27	1.06	1.05	1.03	1.00
$SP_{\varsigma,\kappa,\gamma}$, $\alpha = 0.001$	1.04	2.06	2.54	3.44	3.85	0.77	0.90	1.04	1.62	2.03	1.07	0.97	1.03	1.47	1.82	1.51	1.15	1.30	2.19	2.80
$SP_{\varsigma,\kappa,\gamma}$, $\alpha = 0.95$	1.00	1.69	1.78	1.76	1.73	0.75	0.86	0.93	1.07	1.11	0.97	0.94	0.98	1.06	1.08	1.47	1.06	1.05	1.14	1.23
$SP_{\varsigma,\kappa,\gamma}$, $\alpha = 1$	0.99	1.27	1.27	1.17	1.14	0.71	0.96	1.02	1.11	1.09	0.94	1.02	1.05	1.12	1.08	1.44	1.04	1.02	1.01	1.04
$SSP_{\varsigma,\kappa}$, $\alpha = 0.001$	4.14	3.34	3.08	2.79	2.74	1.00	1.07	1.12	1.31	1.43	0.95	1.02	1.05	1.17	1.26	1.14	1.21	1.27	1.62	1.84
$SSP_{\varsigma,\kappa}$, $\alpha = 0.95$	2.00	1.84	1.80	1.74	1.71	0.81	0.92	0.97	1.09	1.13	0.87	0.96	0.99	1.06	1.08	1.07	1.06	1.08	1.19	1.29
$SSP_{\varsigma,\kappa}$, $\alpha = 1$	1.61	1.40	1.38	1.25	1.21	1.04	1.11	1.09	1.16	1.20	1.09	1.07	1.06	1.07	1.16	1.06	1.06	1.09	0.99	0.97
KF-MCMC	4.48	3.47	3.14	2.56	2.33	1.01	1.05	1.07	1.14	1.16	0.88	0.90	0.91	0.94	0.96	0.94	1.01	1.07	1.26	1.35
KF-MCMC-SV	3.61	2.84	2.59	2.21	2.09	0.84	0.89	0.92	1.02	1.07	0.78	0.80	0.81	0.87	0.91	0.80	0.88	0.94	1.17	1.30
ChagePoint 0.2%	2.83	2.37	1.41	1.16	1.26	0.82	0.68	0.77	0.79	1.05	2.87	1.93	1.60	1.15	1.26	1.95	1.38	1.13	1.02	1.16
ChagePoint 2%	4.56	3.60	4.29	3.52	3.34	1.33	1.54	1.78	1.84	1.99	1.35	1.44	1.57	1.81	1.89	1.79	1.91	1.96	1.76	2.82
ChagePoint 10%	11.73	8.65	8.73	7.34	6.21	2.47	2.94	2.99	3.17	3.17	2.20	2.31	2.42	2.66	2.56	2.74	3.20	2.97	2.90	3.44

Table 4: Monte Carlo with 10 regressors. Table reports the 1-step ahead absolute parameter distance relative to that of the Kalman Filter of several estimators of TVP models. The considered estimators are the following: 1) OLS; 2) forgetting factor with constant parameters (CFF); 3) Forgetting factor with the dynamic selection of λ and κ (KK), with $\lambda \in [0.9, 0.91, ..., 0.99]$ and $\kappa \in [0.94, 0.96, 0.98]$ as in Koop and Korobilis (2013); 4) the self-perturbed Kalman filter of Park (1992) (SP) with dynamic selection of $\varsigma, \kappa, \gamma$ with $\varsigma \in [0.01, 0.02, 0.03, 0.04]$, $\kappa \in [0.94, 0.96, 0.98]$ and $\gamma \in [0.01, 0.21, 0.41, 0.61, 0.81, 1.01, 1.21, 1.41]$; 5) the standardized self-perturbed Kalman filter, (SSP), with dynamic selection of ς, κ with $\varsigma \in [0.01, 0.02, 0.03, 0.04]$ and $\kappa \in [0.94, 0.96, 0.98]$; 6) MCMC with Kalman Filter for TVP model (KF-MCMC); 7) MCMC with Kalman Filter for TVP model under stochastic volatility (KF-MCMC-SV); 8) Change Point model of Pesaran et al (2006) with different number of breaks percentages. The dynamic selection of the design parameters λ, ς, κ and γ has been performed with DMS for different values of $\alpha \in [0.001, 0.95, 1]$. The model contains $m = 10$ regressors with a sample size of $T = 500$. Last column reports the CPU time relative to that of the Kalman Filter.

	No Breaks					**One Break**					**Three Breaks**					**Random Walk**					**CPU**
	0.1	**0.5**	**1**	**5**	**10**	**0.1**	**0.5**	**1**	**5**	**10**	**0.1**	**0.5**	**1**	**5**	**10**	**0.1**	**0.5**	**1**	**5**	**10**	
iid Gaussian:																					
OLS	0.59	0.60	0.60	0.66	0.70	2.60	1.68	1.38	0.91	0.80	2.47	1.72	1.44	0.99	0.86	1.89	1.33	1.15	0.84	0.75	0.00
CFF-$\lambda = 0.96$, $\kappa = 0.94$	2.05	2.03	2.03	1.86	1.72	1.24	1.22	1.29	1.51	1.62	1.26	1.07	1.06	1.25	1.39	1.09	1.12	1.20	1.51	1.64	0.00
CFF-$\lambda = 0.98$, $\kappa = 0.94$	1.51	1.47	1.46	1.33	1.24	1.47	1.14	1.11	1.15	1.20	1.73	1.19	1.07	1.03	1.07	1.28	1.07	1.06	1.16	1.22	0.00
KK, $\alpha = 0.001$	1.20	1.17	1.15	1.05	0.97	1.55	1.20	1.13	1.04	1.03	1.77	1.29	1.16	1.02	0.99	1.31	1.15	1.10	1.06	1.06	0.02
KK, $\alpha = 0.95$	1.08	1.04	1.02	0.93	0.86	1.45	1.19	1.15	1.05	1.00	1.58	1.21	1.13	1.04	0.99	1.23	1.14	1.11	1.05	1.02	0.02
KK, $\alpha = 1$	1.07	1.04	1.02	0.93	0.86	1.74	1.38	1.29	1.11	1.04	1.69	1.27	1.21	1.08	1.02	1.30	1.24	1.21	1.10	1.05	0.02
$SP_{\varsigma,\kappa,\gamma}$, $\alpha = 0.001$	1.99	2.07	2.09	1.97	1.85	1.13	1.18	1.26	1.55	1.69	1.12	1.06	1.07	1.28	1.43	1.41	1.14	1.18	1.51	1.67	0.28
$SP_{\varsigma,\kappa,\gamma}$, $\alpha = 0.95$	1.10	1.08	1.13	1.08	1.07	1.10	1.10	1.09	1.03	1.02	1.09	1.06	1.05	1.02	1.00	1.41	1.10	1.08	1.05	1.04	0.28
$SP_{\varsigma,\kappa,\gamma}$, $\alpha = 1$	1.04	1.05	1.08	1.04	1.05	1.15	1.15	1.15	1.06	1.04	1.11	1.09	1.08	1.03	1.01	1.42	1.13	1.11	1.06	1.04	0.27
$SSP_{\varsigma,\kappa}$, $\alpha = 0.001$	2.47	2.15	2.05	1.78	1.66	1.24	1.27	1.31	1.45	1.55	1.08	1.09	1.10	1.22	1.32	1.13	1.17	1.23	1.44	1.55	0.04
$SSP_{\varsigma,\kappa}$, $\alpha = 0.95$	1.37	1.12	1.09	1.04	1.02	1.17	1.14	1.11	1.05	1.03	1.09	1.07	1.07	1.04	1.02	1.15	1.11	1.09	1.06	1.04	0.04
$SSP_{\varsigma,\kappa}$, $\alpha = 1$	1.35	1.10	1.06	1.01	0.99	1.33	1.27	1.22	1.04	1.00	1.17	1.17	1.18	1.07	1.00	1.30	1.23	1.17	1.05	1.01	0.04
KF-MCMC	1.92	1.59	1.49	1.24	1.18	1.20	1.22	1.23	1.26	1.26	0.87	0.90	0.92	1.00	1.04	1.04	1.15	1.20	1.32	1.34	1.38
KF-MCMC-SV	1.94	1.71	1.63	1.39	1.27	1.23	1.28	1.31	1.38	1.40	0.99	1.01	1.03	1.13	1.18	1.04	1.14	1.22	1.40	1.45	2.15
ChagePoint 0.2%	0.65	0.87	0.75	0.76	0.84	1.99	1.46	1.31	0.99	0.91	2.38	1.65	1.40	0.99	0.86	1.90	1.14	1.02	0.93	0.93	0.66
ChagePoint 2%	1.57	1.35	1.16	1.06	1.06	2.17	1.54	1.39	1.15	1.08	2.16	1.53	1.36	1.11	1.06	1.65	1.06	0.96	0.85	0.86	0.75
ChagePoint 10%	2.80	2.09	1.75	1.43	1.31	2.92	2.14	1.90	1.60	1.51	2.38	1.79	1.62	1.40	1.35	1.81	1.10	0.94	0.79	0.78	0.85
Student's t(3):																					
OLS	0.63	0.64	0.64	0.64	0.64	1.96	1.23	1.02	0.75	0.70	2.00	1.32	1.12	0.81	0.73	1.52	1.05	0.91	0.71	0.68	0.00
CFF-$\lambda = 0.96$, $\kappa = 0.94$	1.69	1.67	1.66	1.62	1.58	1.11	1.15	1.22	1.40	1.48	1.08	0.99	1.03	1.25	1.37	1.01	1.09	1.18	1.42	1.52	0.00
CFF-$\lambda = 0.98$, $\kappa = 0.94$	1.23	1.19	1.18	1.10	1.04	1.16	0.98	0.98	1.04	1.07	1.36	0.99	0.94	0.96	1.01	1.09	0.96	0.96	1.05	1.10	0.00
KK, $\alpha = 0.001$	1.00	0.95	0.94	0.81	0.72	1.25	1.03	0.97	0.91	0.89	1.44	1.10	1.02	0.91	0.88	1.17	1.03	0.98	0.93	0.93	0.03
KK, $\alpha = 0.95$	0.87	0.83	0.82	0.71	0.62	1.21	1.04	0.98	0.88	0.83	1.31	1.07	1.02	0.90	0.84	1.12	1.03	0.99	0.89	0.86	0.03
KK, $\alpha = 1$	0.88	0.83	0.82	0.71	0.62	1.43	1.17	1.08	0.91	0.84	1.40	1.15	1.09	0.93	0.85	1.21	1.11	1.05	0.92	0.88	0.03
$SP_{\varsigma,\kappa,\gamma}$, $\alpha = 0.001$	1.79	1.84	1.86	1.89	1.91	1.05	1.13	1.23	1.51	1.63	1.04	1.00	1.05	1.34	1.50	1.09	1.07	1.15	1.48	1.63	0.49
$SP_{\varsigma,\kappa,\gamma}$, $\alpha = 0.95$	0.98	0.96	0.94	0.94	1.00	1.03	0.97	0.95	0.91	0.92	1.03	0.97	0.96	0.91	0.91	1.08	0.98	0.95	0.92	0.93	0.47
$SP_{\varsigma,\kappa,\gamma}$, $\alpha = 1$	0.95	0.91	0.88	0.92	1.00	1.09	1.04	0.98	0.92	0.91	1.06	1.02	0.99	0.91	0.90	1.12	1.01	0.96	0.91	0.92	0.47
$SSP_{\varsigma,\kappa}$, $\alpha = 0.001$	1.88	1.66	1.60	1.55	1.53	1.16	1.18	1.21	1.32	1.38	1.05	1.03	1.05	1.19	1.28	1.09	1.13	1.16	1.31	1.39	0.05
$SSP_{\varsigma,\kappa}$, $\alpha = 0.95$	1.03	0.92	0.90	0.87	0.86	1.07	1.00	0.96	0.89	0.88	1.04	0.99	0.97	0.91	0.88	1.07	1.00	0.96	0.90	0.90	0.05
$SSP_{\varsigma,\kappa}$, $\alpha = 1$	1.02	0.89	0.87	0.85	0.85	1.21	1.07	0.98	0.88	0.86	1.13	1.13	1.08	0.90	0.87	1.21	1.05	0.97	0.87	0.87	0.05
KF-MCMC	1.58	1.34	1.27	1.14	1.13	1.19	1.21	1.22	1.23	1.21	0.88	0.91	0.94	1.04	1.07	1.08	1.19	1.23	1.29	1.29	2.17
KF-MCMC-SV	1.37	1.19	1.11	1.01	0.96	1.13	1.14	1.14	1.15	1.14	0.95	0.94	0.94	0.98	1.03	1.00	1.08	1.13	1.19	1.21	3.40
ChagePoint 0.2%	1.16	1.29	1.39	1.13	0.83	1.65	1.20	1.08	0.90	0.87	1.96	1.28	1.12	0.80	0.71	1.18	0.90	0.82	0.70	0.59	1.03
ChagePoint 2%	1.74	1.68	1.62	1.16	0.83	1.82	1.38	1.23	1.05	0.93	1.73	1.27	1.16	0.98	0.90	1.03	0.75	0.69	0.59	0.49	1.27
ChagePoint 10%	2.76	2.07	1.90	1.17	0.83	2.36	1.80	1.63	1.39	1.26	1.99	1.53	1.41	1.24	1.10	1.08	0.75	0.65	0.51	0.42	1.41
GARCH(1,1):																					
OLS	0.59	0.61	0.61	0.65	0.71	2.61	1.70	1.39	0.93	0.81	2.48	1.71	1.45	1.00	0.87	1.88	1.34	1.15	0.83	0.75	0.00
CFF-$\lambda = 0.96$, $\kappa = 0.94$	2.02	2.02	2.01	1.88	1.72	1.25	1.22	1.28	1.49	1.60	1.27	1.06	1.05	1.23	1.36	1.09	1.10	1.18	1.46	1.60	0.00
CFF-$\lambda = 0.98$, $\kappa = 0.94$	1.49	1.47	1.45	1.33	1.25	1.49	1.14	1.10	1.14	1.19	1.74	1.19	1.07	1.02	1.07	1.28	1.06	1.04	1.12	1.19	0.00
KK, $\alpha = 0.001$	1.20	1.19	1.17	1.05	0.99	1.56	1.21	1.13	1.04	1.02	1.78	1.29	1.17	1.02	0.99	1.32	1.14	1.09	1.04	1.04	0.03
KK, $\alpha = 0.95$	1.07	1.06	1.04	0.92	0.89	1.48	1.20	1.15	1.05	0.99	1.60	1.21	1.14	1.04	0.99	1.24	1.13	1.12	1.05	1.01	0.03
KK, $\alpha = 1$	1.07	1.06	1.05	0.92	0.88	1.74	1.39	1.30	1.10	1.02	1.72	1.28	1.22	1.09	1.03	1.30	1.23	1.22	1.11	1.05	0.03
$SP_{\varsigma,\kappa,\gamma}$, $\alpha = 0.001$	2.03	2.13	2.15	2.05	1.89	1.13	1.21	1.29	1.58	1.73	1.13	1.06	1.07	1.28	1.45	1.41	1.13	1.17	1.51	1.67	0.48
$SP_{\varsigma,\kappa,\gamma}$, $\alpha = 0.95$	1.14	1.10	1.15	1.10	1.09	1.12	1.12	1.11	1.04	1.03	1.10	1.05	1.05	1.02	1.01	1.42	1.11	1.09	1.03	1.01	0.47
$SP_{\varsigma,\kappa,\gamma}$, $\alpha = 1$	1.03	1.06	1.09	1.03	1.05	1.16	1.16	1.15	1.06	1.03	1.12	1.08	1.08	1.03	1.01	1.42	1.13	1.12	1.04	1.00	0.46
$SSP_{\varsigma,\kappa}$, $\alpha = 0.001$	2.53	2.23	2.12	1.86	1.72	1.26	1.31	1.33	1.49	1.58	1.09	1.09	1.11	1.24	1.36	1.14	1.19	1.24	1.44	1.55	0.05
$SSP_{\varsigma,\kappa}$, $\alpha = 0.95$	1.37	1.14	1.12	1.05	1.04	1.17	1.15	1.13	1.06	1.03	1.09	1.06	1.06	1.04	1.02	1.15	1.10	1.09	1.04	1.02	0.05
$SSP_{\varsigma,\kappa}$, $\alpha = 1$	1.35	1.12	1.07	0.99	0.99	1.33	1.28	1.22	1.04	0.99	1.17	1.15	1.17	1.06	1.00	1.29	1.22	1.16	1.02	0.98	0.05
KF-MCMC	1.98	1.78	1.69	1.42	1.31	1.24	1.30	1.33	1.40	1.42	0.99	1.00	1.03	1.12	1.18	1.05	1.16	1.23	1.40	1.44	2.18
KF-MCMC-SV	1.93	1.62	1.51	1.28	1.20	1.20	1.23	1.24	1.27	1.27	0.86	0.88	0.90	0.98	1.03	1.04	1.14	1.19	1.29	1.32	3.40
ChagePoint 0.2%	0.65	0.66	0.72	0.83	0.95	2.05	1.49	1.32	1.02	0.93	2.39	1.65	1.40	0.99	0.88	1.92	1.12	1.01	0.94	0.94	1.04
ChagePoint 2%	1.53	1.25	1.29	1.14	1.12	2.30	1.62	1.41	1.18	1.08	2.08	1.53	1.35	1.12	1.07	1.70	1.02	0.94	0.87	0.84	1.35
ChagePoint 10%	2.90	2.03	1.97	1.50	1.38	2.91	2.14	1.88	1.55	1.52	2.36	1.79	1.64	1.39	1.39	1.94	1.07	0.93	0.79	0.76	1.53

Figure 1: Parameter estimates for the model with one break and noise-to-signal ration $\sigma = 1$. The two panels report the estimates of the true parameters (solid black lines) together with the estimates obtained with change point model of Pesaran et al. (2006) (dashed-red line) with 0.2% of shifts, and standard Kalman filter (purple-dotted line).