Appendix 4: Derivation of the asymptotic standard errors using the Delta Method

Let the kurtosis coefficient of student t is $Kurt = 3\left(\frac{\nu-2}{\nu-4}\right)$, we have the variance of kurtosis coefficient by delta approach based on t distribution is $Var(Kurt) = \left(\frac{\partial Kurt}{\partial \nu}\right)^2 Var(\nu)$, where õ is the estimated degree of freedom from (E)GARCH-t model.

The skewness coefficient of EGB2 distribution is given by

$$skew-egb2 = g(p,q) = \frac{\psi'(p)-\psi'(q)}{[\psi'(p)+\psi'(q)]^{1.5}}$$

The variance of skewness coefficient by delta approach based on EGB2 distribution is

$$var(skew-egb2) = \left[g'_p(p,q)\right]^2 var(p) + \left[g'_q(p,q)\right]^2 var(q) + 2g'_p(p,q)g'_q(p,q)cov(p,q)$$

where

$$g'_p(p,q) = \frac{\psi'''(p)[\psi'(p)+\psi'(p)]^{1.5} - 1.5\psi''(p)[\psi'(p)+\psi'(p)]^{0.5}[\psi''(p)-\psi''(q)]}{[\psi'(p)+\psi'(p)]^3}$$

$$g'_q(p,q) = \frac{\psi'''(q)[\psi'(p)+\psi'(q)]^{1.5} + 1.5\psi''(q)[\psi'(p)+\psi'(q)]^{0.5}[\psi''(p)-\psi''(q)]}{[\psi'(p)+\psi'(q)]^3}$$

The kurtosis coefficient of EGB2 distribution is given by

$$kurt\_egb2 = h(p,q) = \frac{\psi'''(p)+\psi'''(q)}{[\psi'(p)+\psi'(q)]^2} + 3$$

The variance of kurtosis coefficient by delta approach based on EGB2 distribution is

$$\mathrm{var}(kurt-egb2) = \left[h'_p(p,q)\right]^2 \mathrm{var}(p) + \left[h'_q(p,q)\right]^2 \mathrm{var}(q) + 2h'_p(p,q)h'_q(p,q)\,\mathrm{cov}(p,q)$$

where $$h'_p(p,q) = \frac{\cdot'''(p)\left[\cdot'(p)+\cdot'(q)\right]^2 - 2\cdot''(p)\left[\cdot'(p)+\cdot'(q)\right]\left[\cdot'''(p)+\cdot'''(q)\right]}{\left[\cdot'(p)+\cdot'(q)\right]^4}$$

and $$h'_q(p,q) = \frac{\cdot'''(q)\left[\cdot'(p)+\cdot'(q)\right]^2 - 2\cdot''(q)\left[\cdot'(p)+\cdot'(q)\right]\left[\cdot'''(p)+\cdot'''(q)\right]}{\left[\cdot'(p)+\cdot'(q)\right]^4}$$